AP Calculus (BC)
Summer Assignment

Name ______________________________

Happy Summer AP Calculus (BC) students-to-be;

This summer packet includes practice problems of skills and concepts you have been introduced to and have worked with in your previous math courses. The purpose of this summer packet is to refresh your memory about concepts, which by the end of the summer may seem a bit distant, so that in September we can continue to move forward at a good pace. I recommend that you start your work mid-August as it must be completed and turned in at the beginning of our first day of class. You must show all of your work to receive credit, so be sure to attach this to your packet prior to turning it in. I will not accept this packet late or if it is incomplete, so please manage your time wisely and plan accordingly. This work should be done without the aid of a calculator (this means exact answers only), however, I encourage you to use one to check your work after it is completed. You will be given the entire first class to ask questions for clarification and for review of what is in this packet. During our second class you will be tested on the material you have revisited in this summer packet and only this material. This Test will represent the first major grade of the first quarter and is not one that can be readdressed to earn back points. If you are not completely solid with these foundational skills it is an indicator that you may have difficulty meeting with success in this rigorous course. If you need to contact me, feel free to email me at cflanders@mvyps.org. Enjoy your summer and I'll be looking forward to seeing you in September.

Mrs. Flanders
MVRHS
Math Department Chair

Topics for Review:

* Transformations and Analysis of Parent Functions
  (Linear, Quadratic, Cubic, Square Root, Cube Root, Absolute Value, Reciprocal and Inverse Square)

* Piecewise Functions

* Trigonometric Functions
  (sine, cosine, tangent, cosecant, secant, cotangent, inverse Sine, inverse Cosine and inverse Tangent)

* Exponential and Logarithmic Functions
  (Transformations and analysis of Parents, Solving Equations)

* Higher Degree Polynomials and Rational Functions
  (Graphing and analysis)

* Operations, Composition and Inverses of Functions

* Difference Quotient

* Unit 1 nba of the AP Calculus Course Curriculum
  (Limits and Continuity)

* Unit 2 of the AP Calculus Course Curriculum
  (Differentiation: Definition and Basic Derivative Rules)

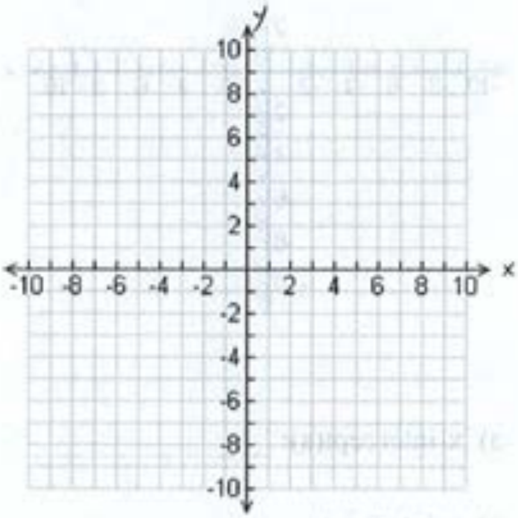
* Chain Rule

Transformations and Analysis of Parent Functions

(For problems 1-16)

Sketch the graph of each and complete the analysis.

1. $y - 5 = 3(x - 2)$



a) x-intercept(s): ______________

b) y-intercept: ______________

c) Domain: ______________

d) Range: ______________

e) Even/Odd or Neither? _________

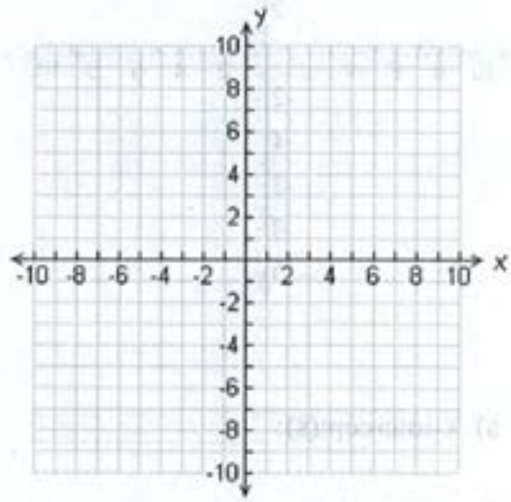
f) Interval of Increase: __________

g) Interval of Decrease: __________

h) End Behavior:

$as\ x \to -\infty \quad y \to$
$as\ x \to +\infty \quad y \to$

2. $x + 6y = 6$



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b) y-intercept: ______________

c) Domain: ______________

d) Range: ______________

e) Even/Odd or Neither? _________

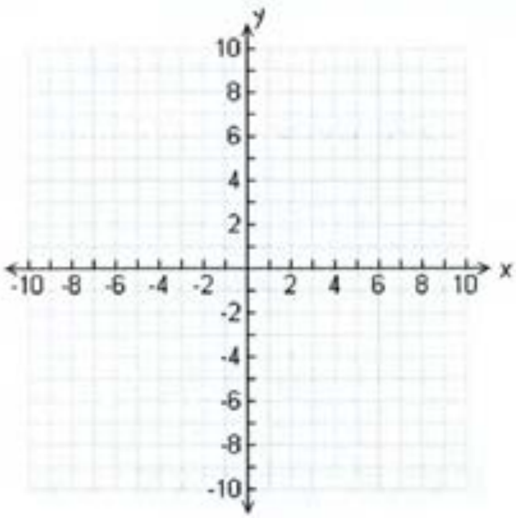
f) Interval of Increase: __________

g) Interval of Decrease: __________

h) End Behavior:

$as\ x \to -\infty \quad y \to$
$as\ x \to +\infty \quad y \to$

3. $y = (x + 4)^2 + 3$



a) x-intercept(s): ______________

b) y-intercept: ______________

c) Domain: ______________

d) Range: ______________

e) Even/Odd or Neither? ________

f) Interval of Increase: _________

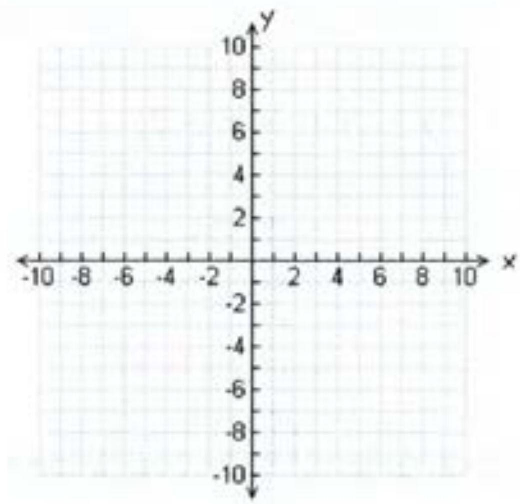
g) Interval of Decrease: _________

h) End Behavior:

$as\ x \rightarrow -\infty \quad y \rightarrow$

$as\ x \rightarrow +\infty \quad y \rightarrow$

4. $y = \frac{-1}{2}(x - 5)^2 - 1$



a) x-intercept(s): ______________

b) y-intercept: ______________

c) Domain: ______________

d) Range: ______________

e) Even/Odd or Neither? ________

f) Interval of Increase: _________

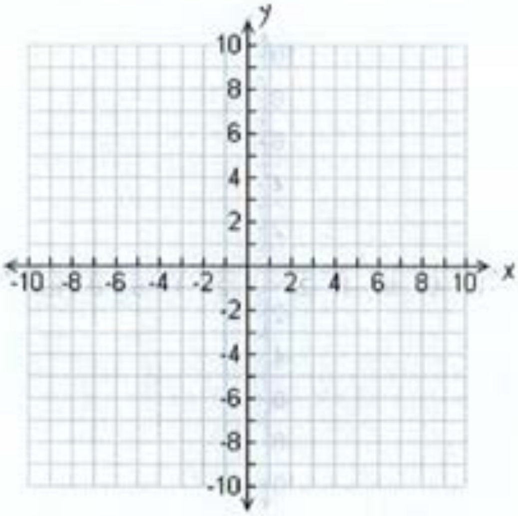
g) Interval of Decrease: _________

h) End Behavior:

$as\ x \rightarrow -\infty \quad y \rightarrow$

$as\ x \rightarrow +\infty \quad y \rightarrow$

5. $y = (x - 4)^3 - 6$



a) x-intercept(s): ____________

b) y-intercept: ____________

c) Domain: ____________

d) Range: ____________

e) Even/Odd or Neither? ________

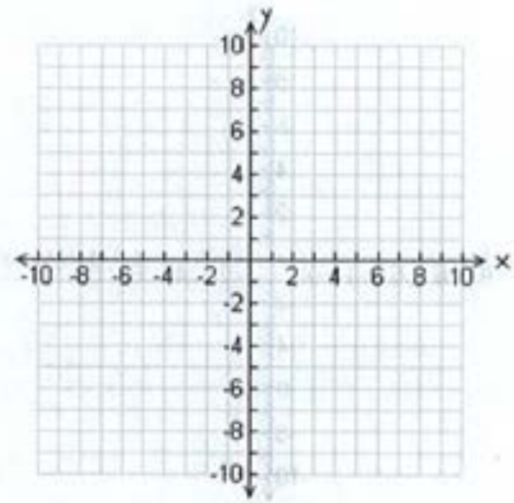
f) Interval of Increase: ________

g) Interval of Decrease: ________

h) End Behavior:

$as\ x \to -\infty \quad y \to$
$as\ x \to +\infty \quad y \to$

6. $y = -(x + 2)^3 + 1$



a) x-intercept(s): ____________

b) y-intercept: ____________

c) Domain: ____________

d) Range: ____________

e) Even/Odd or Neither? ________

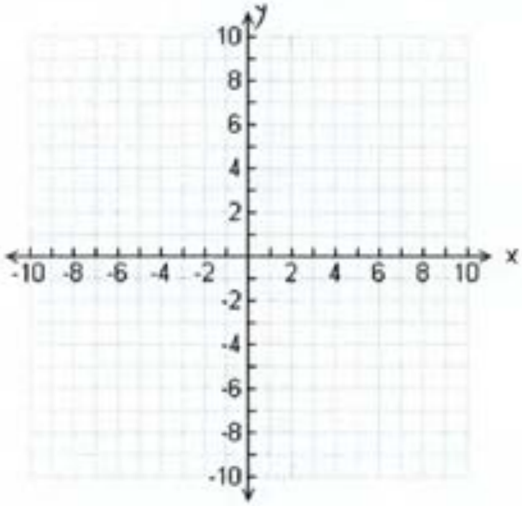
f) Interval of Increase: ________

g) Interval of Decrease: ________

h) End Behavior:

$as\ x \to -\infty \quad y \to$
$as\ x \to +\infty \quad y \to$

7. $y = -\sqrt{x + 3}$



a) x-intercept(s): _______________

b) y-intercept: _______________

c) Domain: _______________

d) Range: _______________

e) Even/Odd or Neither? _________

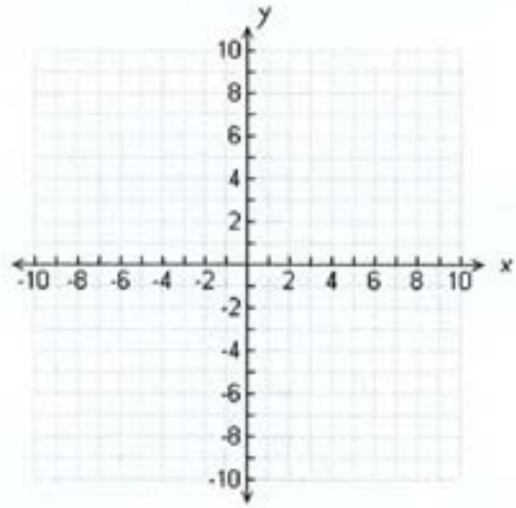
f) Interval of Increase: __________

g) Interval of Decrease: __________

h) End Behavior:

$as\ x \rightarrow \qquad y \rightarrow$

8. $y = \sqrt{-x + 2} - 4$



a) x-intercept(s): _______________

b) y-intercept: _______________

c) Domain: _______________

d) Range: _______________

e) Even/Odd or Neither? _________

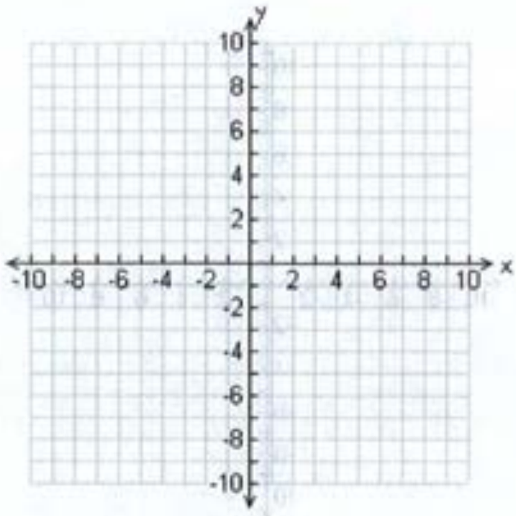
f) Interval of Increase: __________

g) Interval of Decrease: __________

h) End Behavior:

$as\ x \rightarrow \qquad y \rightarrow$

9. $y = \sqrt[3]{x + 2} - 1$



a) x-intercept(s): ____________

b) y-intercept: ____________

c) Domain: ____________

d) Range: ____________

e) Even/Odd or Neither? ____________

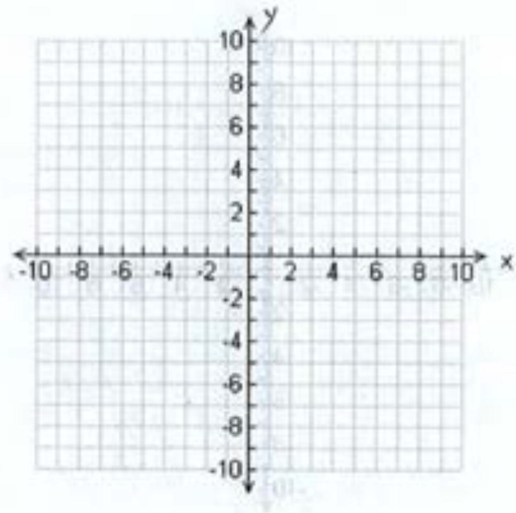
f) Interval of Increase: ____________

g) Interval of Decrease: ____________

h) End Behavior:

$as\ x \to -\infty\ \ y \to$
$as\ x \to +\infty\ \ y \to$

10. $y = -2\sqrt[3]{x} + 5$



a) x-intercept(s): ____________

b) y-intercept: ____________

c) Domain: ____________

d) Range: ____________

e) Even/Odd or Neither? ____________

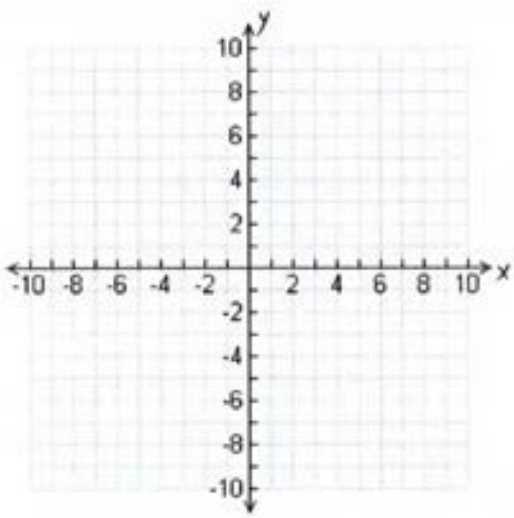
f) Interval of Increase: ____________

g) Interval of Decrease: ____________

h) End Behavior:

$as\ x \to -\infty\ \ y \to$
$as\ x \to +\infty\ \ y \to$

11. $y = 2|x| - 5$



a) x-intercept(s): _______________

b) y-intercept: _______________

c) Domain: _______________

d) Range: _______________

e) Even/Odd or Neither? _________

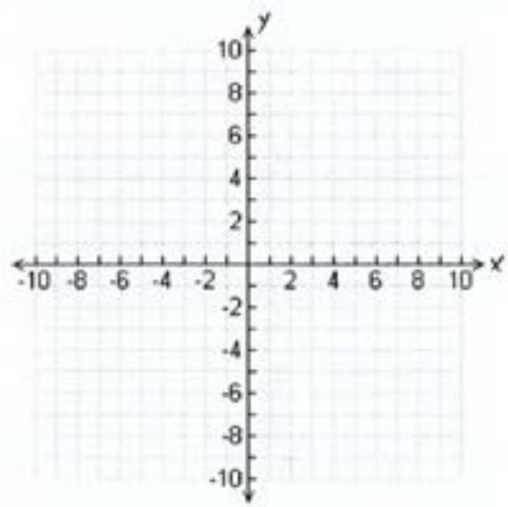
f) Interval of Increase: __________

g) Interval of Decrease: __________

h) End Behavior:

$as\ x \to\ -\infty\quad y \to$
$as\ x \to\ +\infty\quad y \to$

12. $y = \frac{-1}{3}|x + 3| + 1$



a) x-intercept(s): _______________

b) y-intercept: _______________

c) Domain: _______________

d) Range: _______________

e) Even/Odd or Neither? _________

f) Interval of Increase: __________

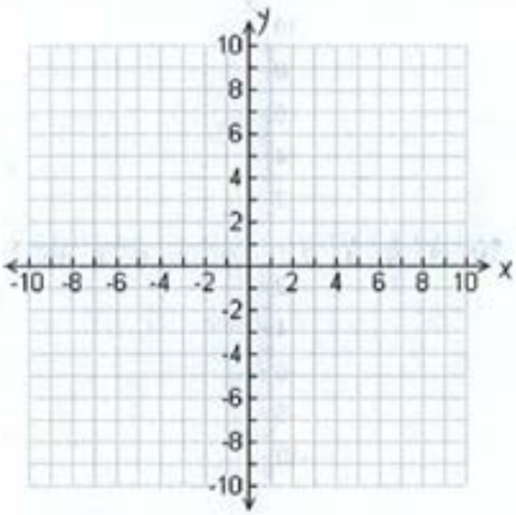
g) Interval of Decrease: __________

h) End Behavior:

$as\ x \to\ -\infty\quad y \to$
$as\ x \to\ +\infty\quad y \to$

(Label aymptotes for 13-16)

13. $y = \frac{3}{x+4} + 2$



a) x-intercept(s): ______________

b) y-intercept: ______________

c) Domain: ______________

d) Range: ______________

e) Even/Odd or Neither? _________

f) Interval of Increase: _________

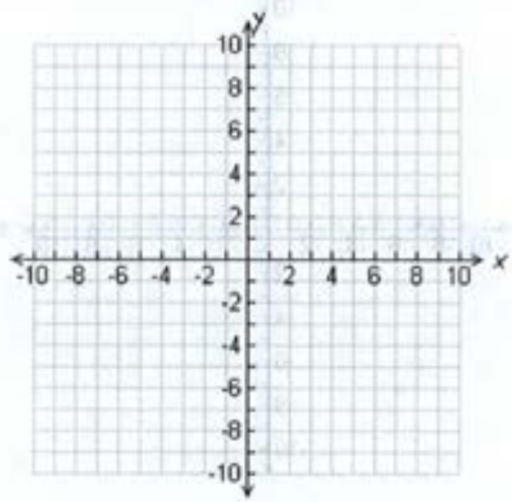
g) Interval of Decrease: _________

h) End Behavior:

$as\ x \to -\infty\ \ y \to$

$as\ x \to +\infty\ \ y \to$

14. $y = \frac{-1}{x}$



a) x-intercept(s): ______________

b) y-intercept: ______________

c) Domain: ______________

d) Range: ______________

e) Even/Odd or Neither? _________

f) Interval of Increase: _________

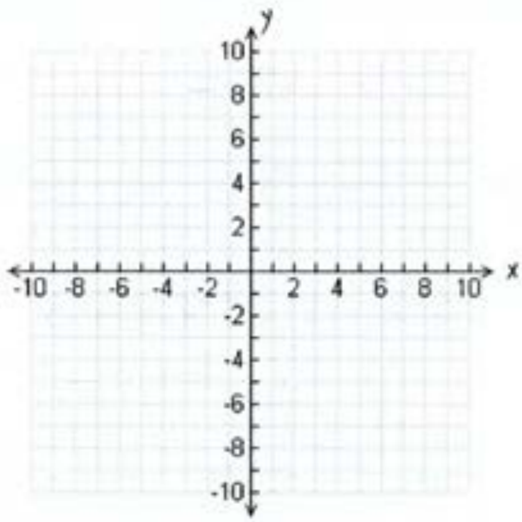
g) Interval of Decrease: _________

h) End Behavior:

$as\ x \to -\infty\ \ y \to$

$as\ x \to +\infty\ \ y \to$

15. $y = \frac{-1}{(x-4)^2} - 3$



a) x-intercept(s): _______________

b) y-intercept: _______________

c) Domain: _______________

d) Range: _______________

e) Even/Odd or Neither? _________

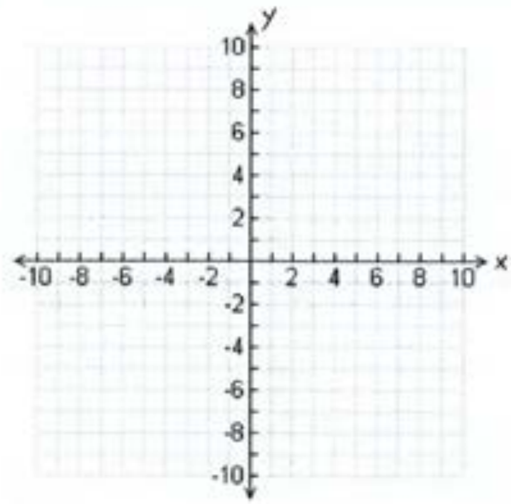
f) Interval of Increase: __________

g) Interval of Decrease: __________

h) End Behavior:

$as\ x \to -\infty \quad y \to$
$as\ x \to +\infty \quad y \to$

16. $y = \frac{2}{(x+3)^2} + 1$



a) x-intercept(s): _______________

b) y-intercept: _______________

c) Domain: _______________

d) Range: _______________

e) Even/Odd or Neither? _________

f) Interval of Increase: __________

g) Interval of Decrease: __________

h) End Behavior:

$as\ x \to -\infty \quad y \to$
$as\ x \to +\infty \quad y \to$

Piecewise Functions

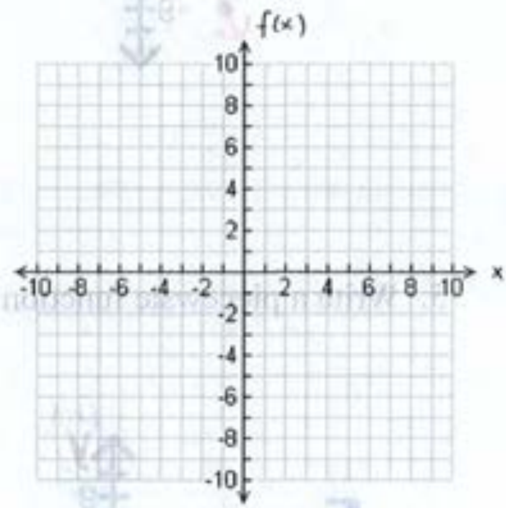
1. Evaluate.

$$f(x)=\begin{cases} x-1 & \text{if} \quad x\le -2 \\ 2x-1 & \text{if} \quad -2<x\le 4 \\ -3x+8 & \text{if} \quad x>4 \end{cases}$$

$f(-1)=$ $\qquad$ $f(-4)=$ $\qquad$ $f(5)=$

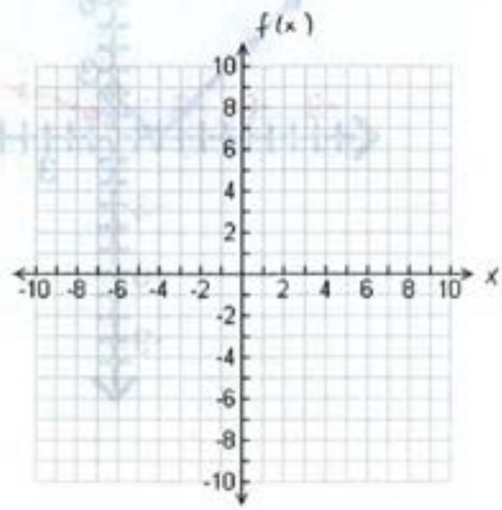
2. Sketch the graph of $f(x)$.

$$f(x)=\begin{cases} -4, & x\le -2 \\ x-2, & -2<x<2 \\ -2x+4, & x\ge 2 \end{cases}$$

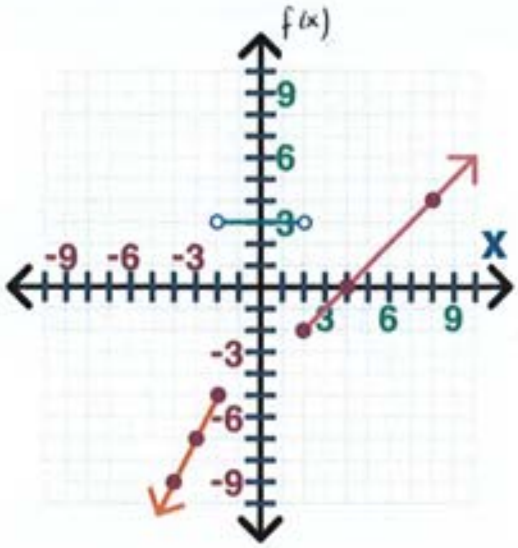


3. Sketch the graph of $f(x)$.

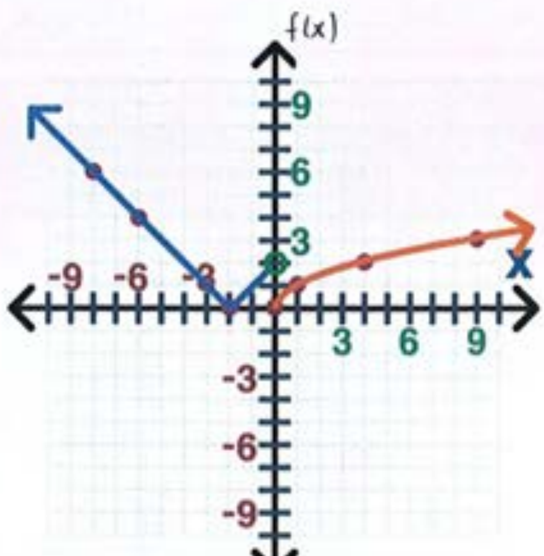
$$f(x)=\begin{cases} 2x+7 & \text{,if } x<-3 \\ x^2+4x+3 & \text{,if } x\ge -3 \end{cases}.$$



4. Write a piecewsie function for $f(x)$.



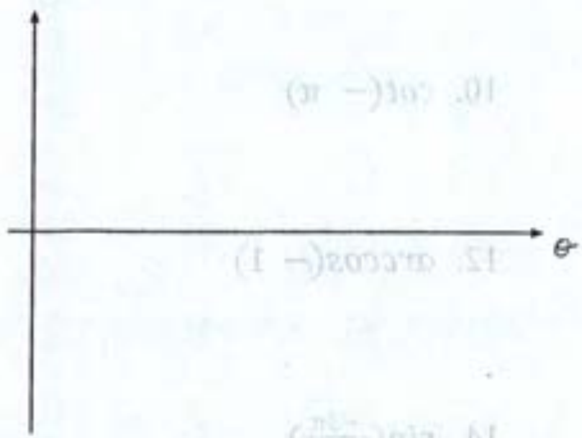
5. Write a piecewsie function for $f(x)$.



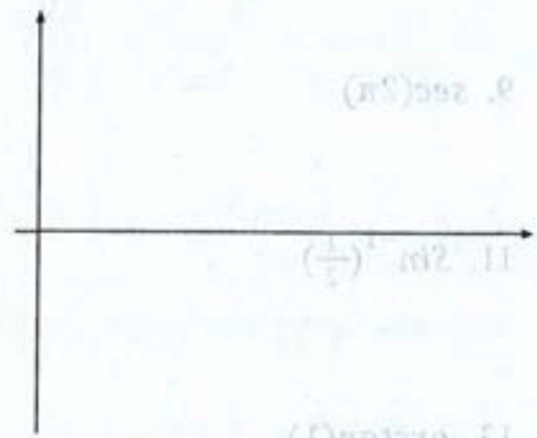
Trigonometric Functions

Sketch at least one period of each of the following. Be sure to label and scale your axes and mark 5 points clearly. (All angle measures are to be in radians)

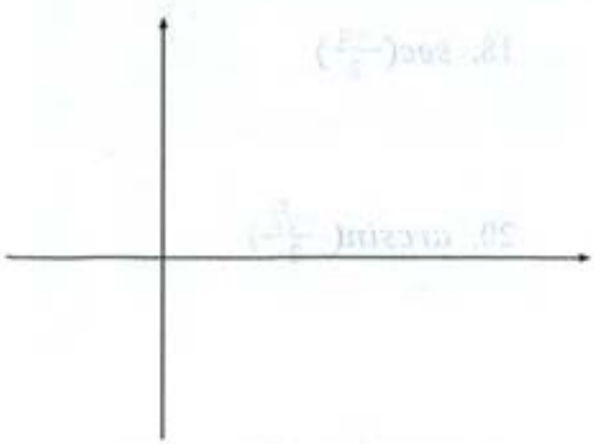
1. $y = 4sin(\theta - \frac{3\pi}{2})$



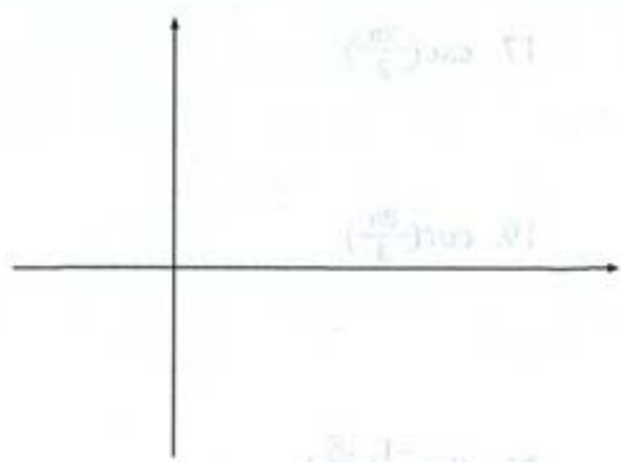
2. $y = 3cos(\theta)$



3. $y =- sin(2\theta - \frac{\pi}{2}) + 3$



4. $y = cos(\frac{\theta}{4} + \frac{\pi}{4}) - 2$



(For problems 5-28)

State the Exact Value or Angle in radians.

5. $sin(\frac{5\pi}{6})$

6. $cos(\frac{5\pi}{3})$

7. $tan(\frac{-\pi}{3})$

8. $csc(\frac{-\pi}{6})$

9. $sec(2\pi)$

10. $cot(-\ \pi)$

11. $Sin^{-1}(\frac{1}{2})$

12. $arccos(-\ 1)$

13. $arctan(1)$

14. $sin(\frac{-3\pi}{4})$

15. $cos(\frac{-3\pi}{4})$

16. $tan(\frac{7\pi}{6})$

17. $csc(\frac{3\pi}{2})$

18. $sec(\frac{-\pi}{2})$

19. $cot(\frac{8\pi}{3})$

20. $arcsin(\frac{-\sqrt{2}}{2})$

21. $Cos^{-1}(\frac{\sqrt{3}}{2})$

22 $Tan^{-1}(-\sqrt{3})$

23. $sin(\frac{11\pi}{3})$

24. $cos(\frac{5\pi}{2})$

25. $tan(\pi)$

26. $csc(\frac{-3\pi}{4})$

27. $sec(\frac{2\pi}{3})$

28. $cot(\frac{7\pi}{6})$

Evaluate.

29. $cot(\frac{\pi}{2}) + 3sin(\frac{3\pi}{2})$

30. $tan(0) - 6sin(\frac{\pi}{2})$

31. $3sec(\pi) - 5tan(4\pi)$

32. $4csc(\frac{3\pi}{2}) + 3cos(\pi)$

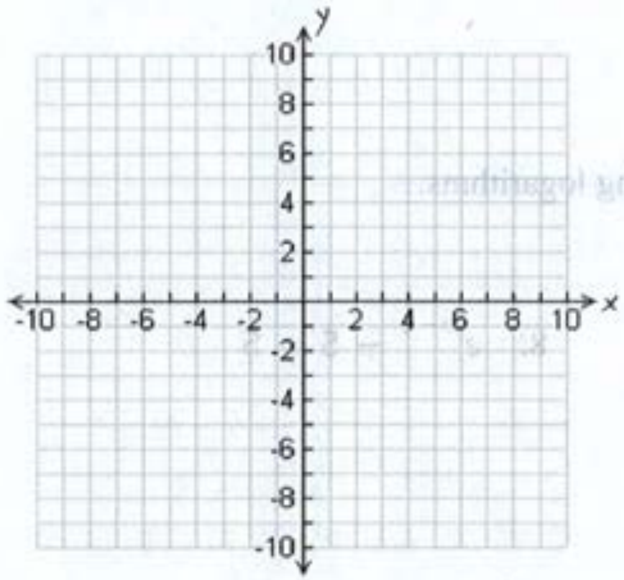
33. $2sec(0) + 4cot^2(\frac{\pi}{2}) + cos(2\pi)$

34. $sin^2(\frac{2\pi}{3}) + cos^2(\frac{2\pi}{3})$

Exponential and Logarithmic Functions

Graph the following exponential equations.

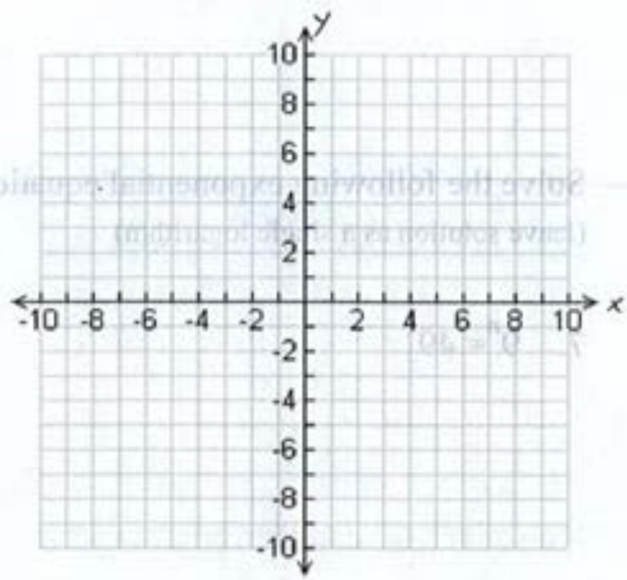
1. $y = 3^{x+1} + 2$



a) Write an equation for the asymptote:

b) State the Domain and Range:

2. $y = -2^{x} - 4$

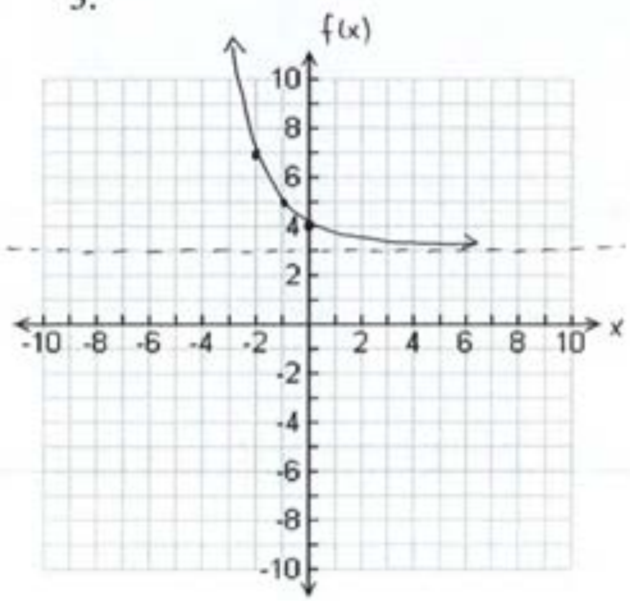


a) Write an equation for the asymptote:

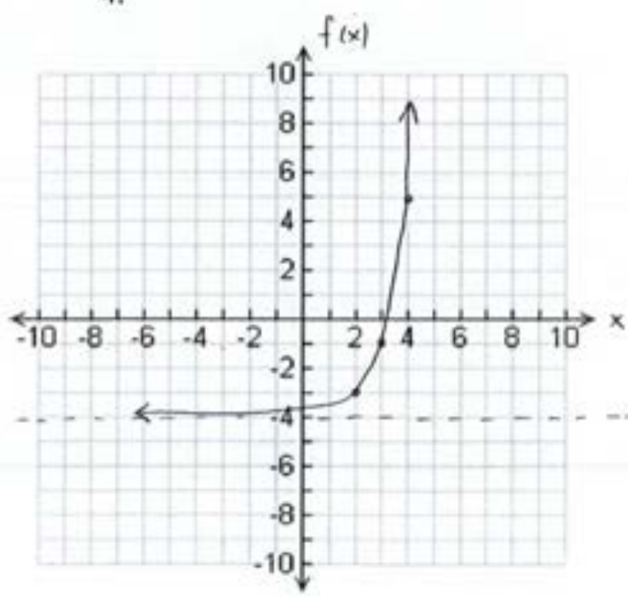
b) State the Domain and Range:

Write an equation for the graphed exponential equations.

3.



4.



Solve the following exponential equations using a common base.

5. $3^{1-2x} = 81$

6. $36^{-2a} = 6^{-5-3a}$

Solve the following exponential equations using logarithms.
(leave solution as a single logarithm)

7. $9^{n} = 49$

8. $e^{x-1} - 5 = 5$

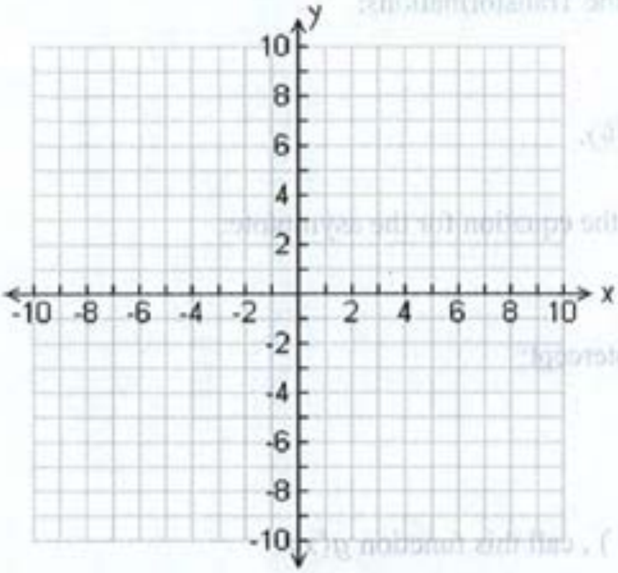
Solve.

9. $2(3^{2x-5}) - 7 = 11$

10. $e^{2x} - 3e^{x} + 2 = 0$

Graph the following logarithmic equations.

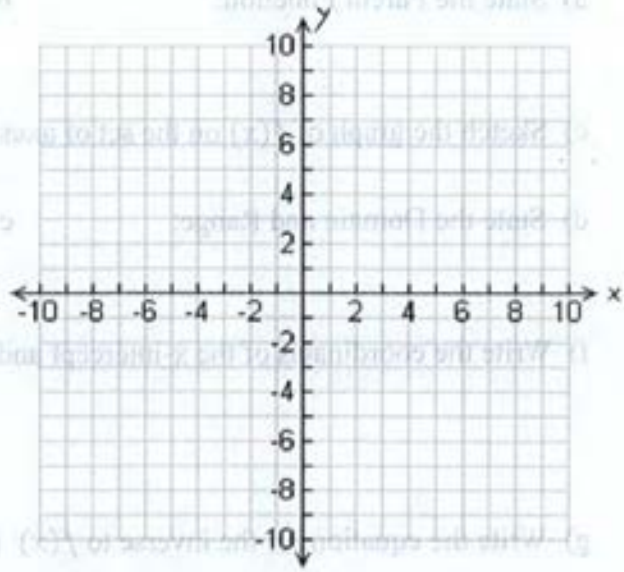
11. $y = log_3(x - 2) - 1$



a) Write an equation for the asymptote:

b) State the Domain and Range:

12. $y = - log_4(x + 3)$

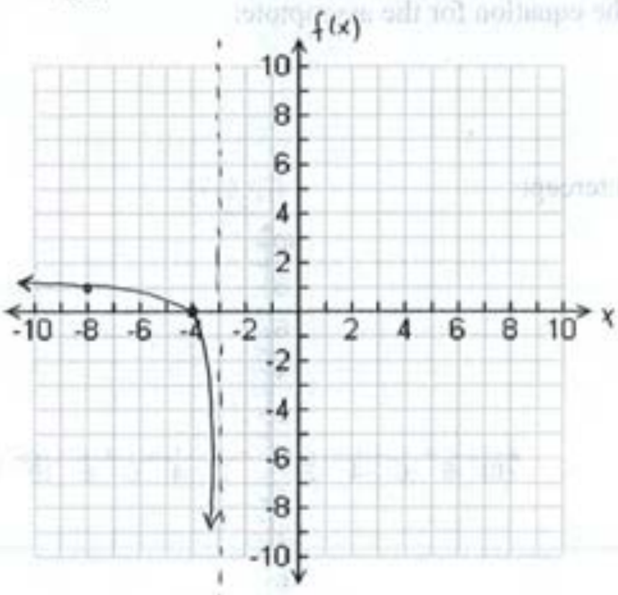


a) Write an equation for the asymptote:

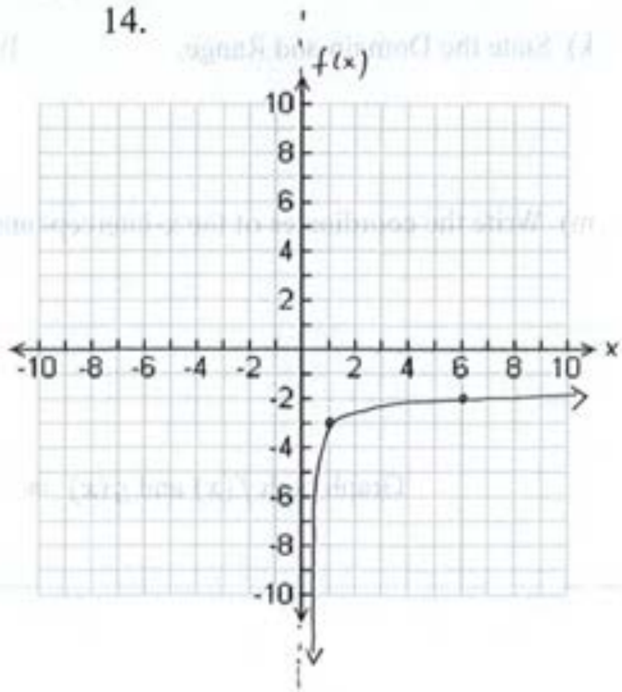
b) State the Domain and Range:

Write an equation for the graphed logarithmic equations.

13.



14.



15. Given: $f(x) = 3^{x+1} - 4$

a) State the Parent Function:

b) State the Transformations:

c) Sketch the graph of $f(x)$ on the set of axes below (⇓).

d) State the Domain and Range:

e) Write the equation for the asymptote:

f) Write the coordinates of the x-intercept and the y-intercept:

g) Write the equation for the inverse to $f(x)$ ($f(x)^{-1}$), call this function $g(x)$

h) State the Parent Function:

i) State the Transformations:

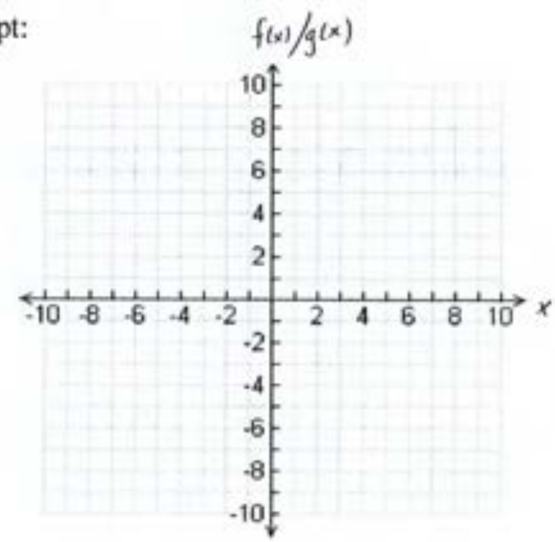
j) Sketch the graph of $g(x)$ on the same set of axes below (⇓).

k) State the Domain and Range:

l) Write the equation for the asymptote:

m) Write the coordinates of the x-intercept and the y-intercept:

Graph both $f(x)$ and $g(x)$ ⇒



Using the properties of logarithms, expand the following.

16. $log_3(x^2y^7)$

Using the properties of logarithms, condense the following.

17. $\frac{1}{2}log_8x$- $3log_8y$

Solve the following logarithmic equations.

18. $log(5x) = log(2x + 9)$

19. $-2log_5(7x) = 2$

20. $ln\,(x - 3) - ln\,(x - 5) = ln\,(5)$

21. $log_4(x) + log_4(x - 12) = 3$

Higher Degree Polynomials and Rational Functions

1. Given: $P(x) = -3x^4 + 5x^2 + 8$

   a) State the end behavior of P(x) :

   b) State all of the possible rational roots of P(x) :

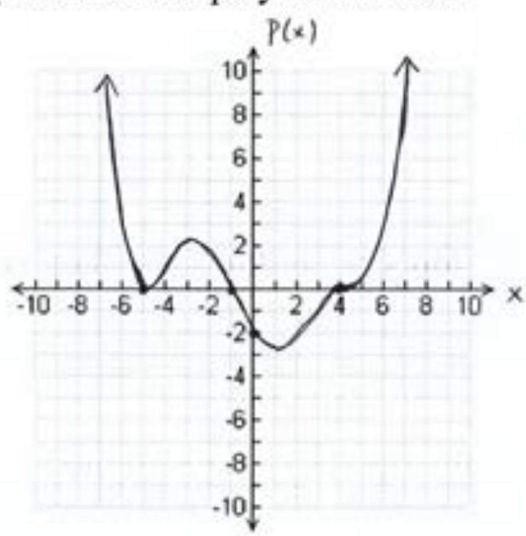
2. Given: $P(x) = 6x^5 - 7x^2 + 1$

   a) At most, how many real zeros will P(x) have?

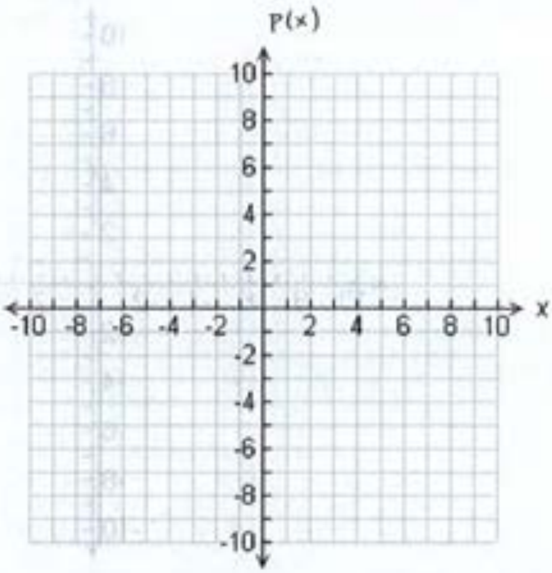
   b) At most, how many extrema will P(x) have?

3. State all of the solutions to $P(x) = x^3 + 6x^2 + 5x - 42$.
   A known factor is $x - 2$.

4. Given the graph below - Write a possible equation for the polynomial based on the intercepts and end behavior:



5. Sketch the graph of a polynomial (P(x)), with a single zero at x = -2, a double zero at x = 0, a double zero at x = 3 and a negative leading coefficient:



(For problems 6 & 7)

Find all of the following (if they exist) and then sketch the graph.

6. $R(x) = \frac{2x^2 - 18}{x^2 + 3x - 4}$

HA'S: __________

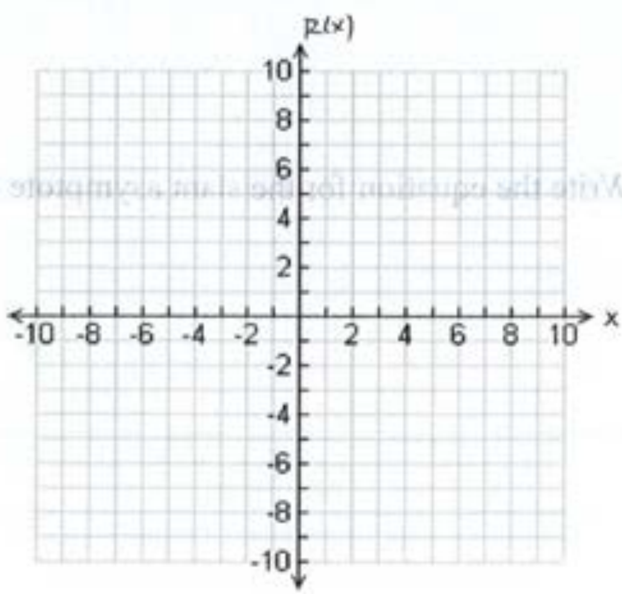
VA'S: __________

Holes: __________

SA'S: __________

X-Int's: __________

Y-Int: __________



7. $R(x) = \dfrac{x^3 - 3x^2 - 6x + 8}{-x^3 + 4x}$

HA'S: __________

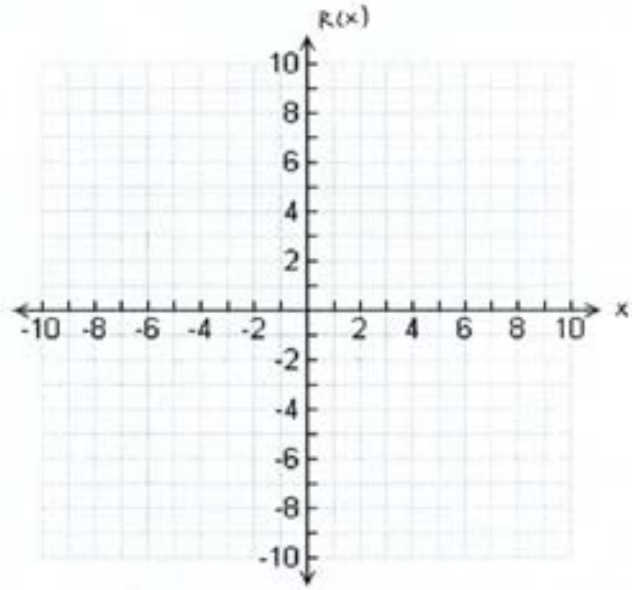
VA'S: __________

Holes: __________

SA'S: __________

X-Int's: __________

Y-Int: __________



8. Write the equation for the slant asymptote to $R(x) = \dfrac{x^3+2x^2-x-2}{x^2+6x+8}$

Operations, Composition and Inverses of Functions

Given $f(x) = 3x^2 - 4x - 5$ and $g(x) = \sqrt{x} + 8$

Find -

1. $f(4)$

2. $g(9)$

3. $f(-3)$

4. $f(1) + g(0)$

5. $g(16) - f(0)$

6. $f(2) \cdot g(25)$

7. $\frac{g(4)}{f(-1)}$

8. $f(a)$

9. $g(2a - 3)$

10. $f(f(a))$

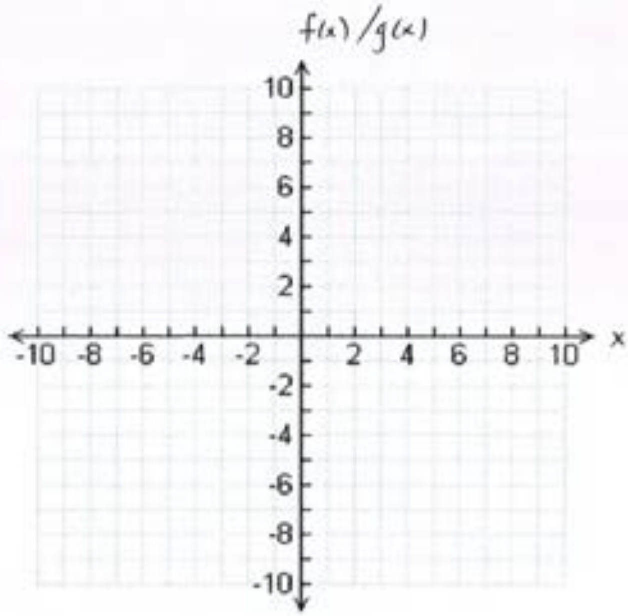
11. $f(x + h)$

12. $g(5 + h)$

Given $f(x) = \frac{2}{3}x - 5$ and $g(x) = 9 + \frac{3}{2}(x - 1)$

Find -

1. $f(3)$
2. $g(-3)$
3. $f(12)$
4. $g(3)$
5. $f(g(9))$
6. $g(f(6))$
7. $f(g(x))$
8. $g(f(x))$
9. Sketch the graphs of both $f(x)$ and $g(x)$ on the same set of axes.



10. What is the relationship between $f(x)$ and $g(x)$?

Difference Quotient

1. Given $P(x) = -5x^2 + 3x - 7$, find $\frac{P(x+h) - P(x)}{h}$

2. Given $P(x) = 4x^3 + 6$, find $\frac{P(x+h) - P(x)}{h}$

3. Given $f(x) = \sqrt{x}$, find $\frac{f(3+h) - f(3)}{h}$

4. Given $f(x) = \sqrt{3x + 2}$, find $\frac{f(x+h) - f(x)}{h}$

5. Given $R(x) = \frac{1}{x}$, find $\frac{R(2+h) - R(2)}{h}$

6. Given $R(x) = \frac{4x}{x-5}$, find $\frac{R(x+h) - R(x)}{h}$

Unit 1 - Limits & Continuity

Determine the limit by substitution.

1) $\lim_{x \to 0} (x^2 - 5)$

2) $\lim_{x \to 2} (x^3 + 5x^2 - 7x + 1)$

3) $\lim_{x \to 0} \frac{x^3 - 6x + 8}{x - 2}$

4) $\lim_{x \to 8} \frac{x^2 + 64}{x + 8}$

5) $\lim_{x \to 5} \sqrt{x^2 + 14x + 49}$

6) $\lim_{x \to 1} (x^2 - 4x)^3$

Determine the limit algebraically, if it exists.

7) $\lim_{x \to 4} \sqrt{x - 6}$

8) $\lim_{x \to 6} \frac{x + 6}{(x - 6)^2}$

9) $\lim_{x \to 7} \frac{|7 - x|}{7 - x}$

10) $\lim_{x \to -6} \frac{x^2 - 36}{x + 6}$

11) $\lim_{x \to -10} \frac{x^2 + 15x + 50}{x + 10}$

12) $\lim_{x \to 0} \frac{x^3 + 12x^2 - 5x}{5x}$

13) $\lim_{x \to 2} \frac{x^2 + 6x - 16}{x - 2}$

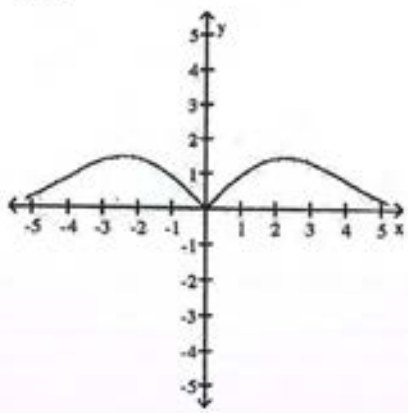
14) $\lim_{x \to 5} \frac{x^2 + 4x - 45}{x^2 - 25}$

15) $\lim_{x \to 0} \frac{\frac{1}{x+8} - \frac{1}{8}}{x}$

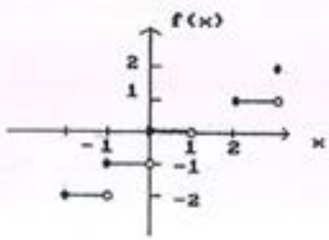
16) $\lim_{x \to 0} \frac{10 \sin x}{6x}$

Determine the limit graphically, if it exists.

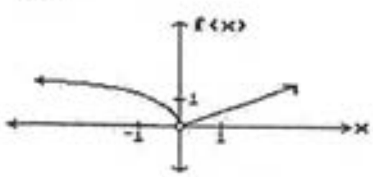
17) $\lim_{x \to 0} f(x)$



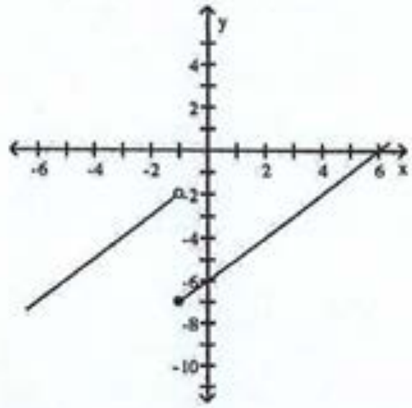
18) $\lim_{x \to -1/2} f(x)$



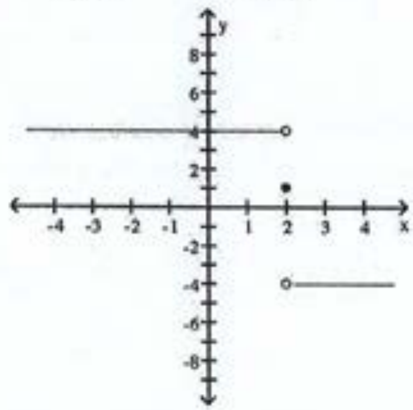
19) $\lim_{x \to 0} f(x)$



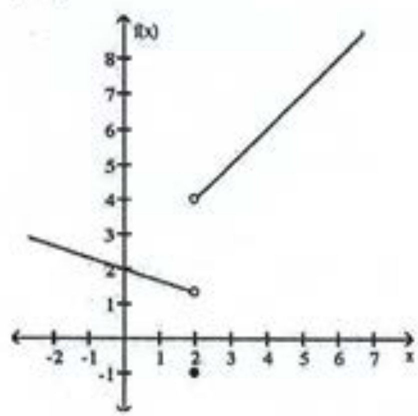
20) Find $\lim_{x \to -1^-} f(x)$ and $\lim_{x \to -1^+} f(x)$.



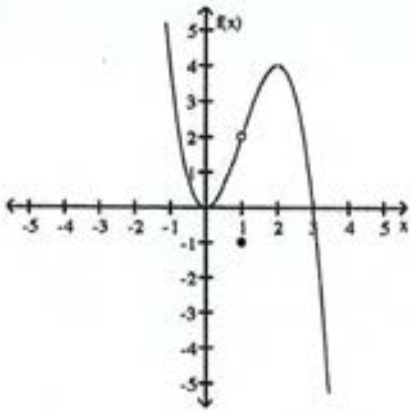
21) Find $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$.



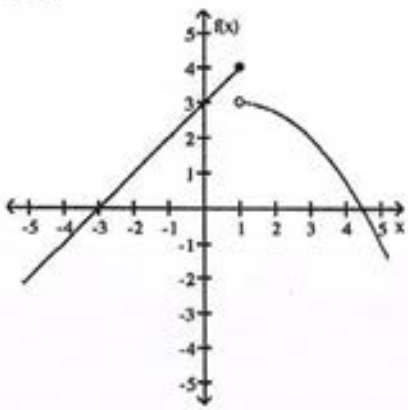
22) $\lim_{x \to 2^-} f(x)$



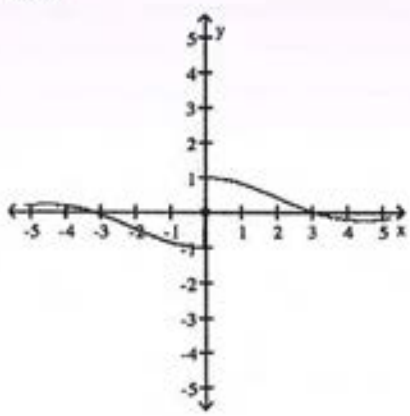
23) $\lim_{x \to 1^-} f(x)$



24) $\lim_{x \to 1^+} f(x)$



25) $\lim_{x \to 0} f(x)$



Find the limit.

26) Let $\lim_{x \to 3} f(x) = -4$ and $\lim_{x \to 3} g(x) = 9$. Find $\lim_{x \to 3} [f(x) - g(x)]$.

27) Let $\lim_{x \to 8} f(x) = -7$ and $\lim_{x \to 8} g(x) = -6$. Find $\lim_{x \to 8} [f(x) \cdot g(x)]$.

28) Let $\lim_{x \to 9} f(x) = 10$ and $\lim_{x \to 9} g(x) = -1$. Find $\lim_{x \to 9} \frac{f(x)}{g(x)}$.

29) Let $\lim_{x \to 9} f(x) = 100$. Find $\lim_{x \to 9} \sqrt{f(x)}$.

30) Let $\lim_{x \to 5} f(x) = -4$ and $\lim_{x \to 5} g(x) = 9$. Find $\lim_{x \to 5} [f(x) + g(x)]^2$.

31) Let $\lim_{x \to -5} f(x) = -7$ and $\lim_{x \to -5} g(x) = 9$. Find $\lim_{x \to -5} \frac{-5f(x) - 9g(x)}{-5 + g(x)}$.

32) Let $\lim_{x \to -6} f(x) = -10$ and $\lim_{x \to -6} g(x) = 7$. Find $\lim_{x \to -6} \frac{[f(x)]^2}{-3 + g(x)}$.

Find the limit, if it exists.

33) $\lim_{x \to \infty} \frac{x^2 - 4x + 14}{x^3 + 9x^2 + 5}$

34) $\lim_{x \to -\infty} \frac{-6 + (2/x)}{7 - (1/x^2)}$

35) $\lim_{x \to -\infty} \frac{-8x^2 + 9x + 5}{-15x^2 + 7x + 14}$

36) $\lim_{x \to -\infty} \frac{\cos 3x}{x}$

37) $\lim_{x \to \infty} \frac{3x + 1}{11x - 7}$

38) $\lim_{x \to \infty} \frac{-5\sqrt{x} + x^{-1}}{-4x + 2}$

39) $\lim_{x \to \infty} \frac{7x^3 - 3x^2 + 3x}{-x^3 - 2x + 7}$

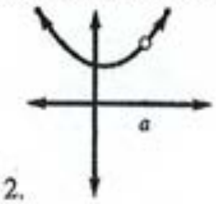
40) $\lim_{x \to \infty} \frac{3x^{-1} + -2x^{-3}}{3x^{-2} + x^{-5}}$

41) $\lim_{x \to -\infty} \frac{5x^3 + 4x^2}{x - 5x^2}$

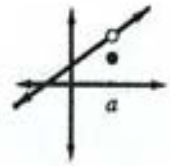
42) $\lim_{x \to -\infty} \frac{\sqrt[3]{x} + 5x + -5}{3x + x^{2/3} + -4}$

Use the definition of continuity to tell why each of the following graphs of $y = f(x)$ is not continuous at $x = a$.

1.



2.



3.

Use the definition of continuity to justify.

4. $h(x) = \begin{cases} 3, & x \le -1 \\ 2ax + b, & -1 < x < 1 \\ -3, & x \ge 1 \end{cases}$ Is h continuous at $x = 1$ if $a = 3$ and $b = -3$? Justify.

5. $f(x) = \begin{cases} x + 3, & x \ne 2 \\ 3, & x = 2 \end{cases}$ Is f continuous at $x = 2$? Justify your answer.

Find a and b so that the function is continuous for all real numbers.

6. $f(x) = \begin{cases} \dfrac{2\sin x}{x}, & x < 0 \\ a - 4x, & x \ge 0 \end{cases}$

8. $h(x) = \begin{cases} 4, & x \le -1 \\ ax + b, & -1 < x < 2 \\ -4, & x \ge 2 \end{cases}$

7. $g(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & x \ne a \\ 6, & x = a \end{cases}$

8. Find a value of k that makes $f(x) = \begin{cases} 2x^2, & x \le \dfrac{1}{2} \\ \sin(kx), & x > \dfrac{1}{2} \end{cases}$ continuous at $x = \dfrac{1}{2}$.

9. If a function f is discontinuous at $x = 2$, which of the following must be true?

I. $\lim_{x \to 2} f(x)$ does not exist

II. $f(2)$ does not exist

III. $\lim_{x \to 2^-} f(x) \ne \lim_{x \to 2^+} f(x)$

IV. $\lim_{x \to 2} f(x) \ne f(2)$

· Using the definition of continuity, determine whether the following functions are continuous over all real numbers.

10. $f(x)=\frac{1}{x^2+1}$	11. $f(x)=\cos\frac{\pi x}{2}$
12. $f(x)=\frac{x+2}{x^2-3x-10}$	13. $f(x)=\frac{\lvert x-3\rvert}{x-3}$
14. $f(x)=\begin{cases}-2x+3, & x<1\\ x^2, & x\geq 1\end{cases}$	15. $f(x)=\begin{cases}-2x, & x\leq 2\\ x^2-4x+1, & x>2\end{cases}$

Intermediate Value Theorem

- For each of the following problems:
  a) Determine if f(x) is continuous over the interval [a, b]
  b) Determine if a root exits over the interval [a, b]
  c) Find the value for c such that f(c) = k over the interval [a, b]

1. $f(x)=x^2+x-1$, [0, 5], $f(c)=11$	2. $f(x)=x^2-6x+8$, [0, 3], $f(c)=0$
3. $f(x)=x^3-x^2+x-2$, [0, 3], $f(c)=4$	4. $f(x)=\frac{x^2+x}{x-1}$, $\left[\frac{5}{2},4\right]$, $f(c)=6$

In the following problems, a function f and a closed interval $[a, b]$ are given. Determine if the Intermediate Value Theorem holds for the given value of k. If the theorem holds, find a number c such that $f(c)=k$. If the theorem does not hold, give the reason.

$y=k$.

5. $f(x)=2+x-x^2$

   $[a, b]=[0, 3]$

   $k=1$

6. $f(x)=\sqrt{9-x^2}$

   $[a, b]=[-2.5, 3]$

   $k=2$

7. $f(x)=\dfrac{1}{x-1}$

   $[a, b]=[2, 5]$

   $k=\dfrac{5}{6}$

Unit 2 - Differentiation

In Exercises 1-3, use the definition

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

to find the derivative of the given function at the indicated point.

1) $f(x)=\frac{1}{x}, a=2$	2) $f(x)=x^2+4, a=1$	3) $f(x)=x^3+x, a=0$

In Exercises 4-6, use the alternate form of the definition

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

to find the derivative of the given function at the indicated point.

4) $f(x)=x^2+4, a=1$	5) $f(x)=\sqrt{x+1}, a=3$	6) $f(x)=2x+3,\ a=-1$

7) For the function $f(x)=\begin{cases}3x^2-4, & x<0\\ 3x-4, & x\geq 0\end{cases}$, determine if $f(x)$ is differentiable at $x=0$.

$$g(x)=\begin{cases}(x+1)^2, & x\leq 0\\ 2x+1\ , & 0<x<5\\ (6-x)^2, & x\geq 5\end{cases}$$

8) Given the function $g(x)$ above:
   a) Determine if $g(x)$ is differentiable at $x=0$
   b) Determine if $g(x)$ is differentiable at $x=5$
   c) State the values of x for which $g(x)$ is differentiable.

Differentiate the following functions. Do not simplify the answer

1) $g(t)=6t^{\frac{3}{5}}$	2) $B(x)=\dfrac{8x^2-6x+11}{x-1}$	3) $f(s)=15-s-4s^2-5s^4$
4) $G(v)=\dfrac{v^3-1}{v^3+1}$	5) $f(x)=3x^2+\sqrt[3]{x^4}$	6) $g(t)=\dfrac{\sqrt[3]{t^2}}{3t-5}$
7) $p(x)=1+\dfrac{1}{x}+\dfrac{1}{x^2}+\dfrac{1}{x^3}$	8) $k(x)=\left(2x^2-4x+1\right)\left(6x-5\right)$	9) $h(x)=x^{\frac{4}{3}}\left(3x^2-2x+5\right)$
10) $M(x)=\dfrac{2x^3-7x^2+4x+3}{x^2}$	11) $f(x)=\dfrac{4x-5}{3x+2}$	12) $f(x)=\dfrac{1}{1+x+x^2+x^3}$

13	Sketch the graph of a continuous function f with $f(0)=-1$ and $f'(x)=\begin{cases}1, & x<-1\\ -2, & x>-1\end{cases}$.
14	**True or False** If $f(x)=x^2+x$, then $f'(x)$ exists for every real number x. Justify your answer.
15	Let $f(x)=4-3x$. Which of the following is equal to $f'(-1)$? (B) 7 (C) -3 (D) 3 (E) does not exist
16	Find the unique value of k that makes the function, $f(x)=\begin{cases}x^3, & x\le 1\\ 3x+k, & x>1\end{cases}$ differentiable at $x=1$.

Chain Rule

Find the derivative of the following functions:

1.

a) $y = 5(2 - x^3)^4$

d) $w(t) = \frac{1}{\sqrt{3t+5}}$

b) $f(x) = \sqrt{x^2 - 4x + 2}$

e) $m(z) = ln(8z^9)$

c) $r(b) = e^{8b^5}$

f) $g(p) = cos(4p^7)$

2. Find the equation of the tangent line to the graph of $y = 3^{sinx} - 4$ when $x = 0$

3. Find $\frac{d}{dx}\, ln(\, lnx\,)$

4. If $h(x) = tan(\,2x\,)$, evaluate $h'(x)$ at $(\,\frac{\pi}{6},\,\sqrt{3}\,)$

5. Find the equation of the normal line to the curve $y = 2tan(\,\frac{\pi x}{4})$ at $x = 1$

6. If $e^{f(x)} = 2 + x^4$, then $f'(x) =$