AP Calculus AB
Summer Assignment
24-25

Name ______________________________

Welcome to AP Calculus!

The following packet includes several questions about concepts you have studied in previous math classes. These concepts will appear in problems throughout the year and will be necessary for you to know in order to be successful in Calculus. The goal of completing this packet is to refresh your memory, so you are ready to start learning in the fall.

This packet is due on our fifth day of class. This is to give you time after school to ask any questions on problems you had trouble answering. I will grade select questions from each section and count this as your first assessment. It is critical that you complete the packet in full and show all your work. Credit will not be given for answers that are not supported by adequate work.

I suggest you start working on this packet mid to late July and do a topic a day. Please do not wait until the last few weeks of summer to complete this packet. You may wish to access videos online, past notes, or the “Things to Know for Calculus” sheet attached to this summer work. I am also available via email to answer questions at kmagnuson@mvrhs.org.

If you struggle to complete this packet and/or you feel like you don’t fully understand these concepts, it is an indicator that you may have difficulty being successful in this course. Please make the appropriate decision whether to continue in AP Calculus or take Honors Calculus.

I look forward to meeting you and having a wonderful year learning Calculus! Please reach out with any questions or concerns.

Ms. Magnuson

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Topic A: Functions

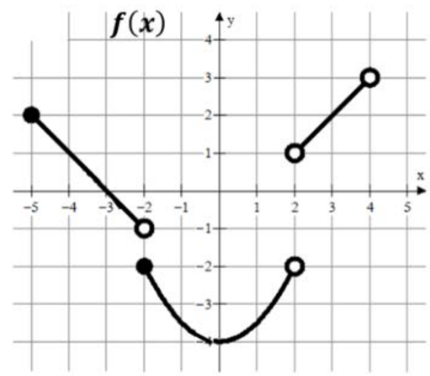
1) If $f(x) = 4x - x^2$, find:

a) $f(4) - f(-4)$

b) $\sqrt{f(\frac{3}{2})}$

c) $\frac{f(x+h)-f(x)}{2h}$

2) Use the graph of $f(x)$ to answer the following. You may estimate.



$f(0) =$

$f(4) =$

$f(-1) =$

$f(-2) =$

$f(2) =$

$f(3) =$

$f(x) = 2$ when $x =$?

$f(x) = -3$ when $x =$?

3) Find the value of each using the piecewise function.

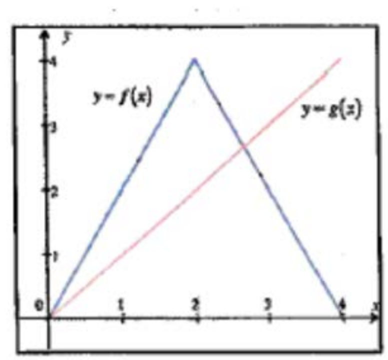
$$f(x) = \begin{cases} -x, & x < 0 \\ x^2 - 1, & 0 \le x < 2 \\ \sqrt{x+2} - 2, & x \ge 2 \end{cases}$$

a) $f(0) - f(2)$

b) $\sqrt{5 - f(-4)}$

c) $f(f(3))$

4) If $f(x)$ and $g(x)$ are given in the graph, find:



a) $(f - g)(3)$

b) $f(g(3))$

Topic B: Linear Functions

5) Write the equation of each line given the slope and a point on the line or two points on the line. . Use point-slope form: $y - y_1 = m(x - x_1)$

a) Slope: 3 Point: (4, -2)

b) (4, -8) and (-3, 12)

c) $f(4) = -8$ and $f(-3) = 12$

6) Find the equation of the lines that are parallel to the given lines.

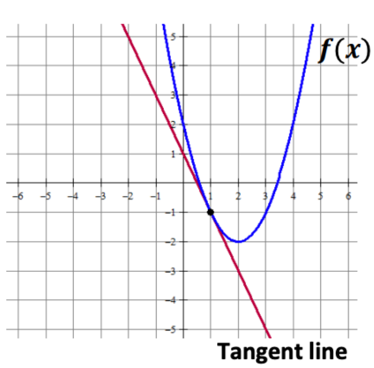
a) (5, -3), $x + y = 3$

b) (-6, 2), $5x + 2y = 7$

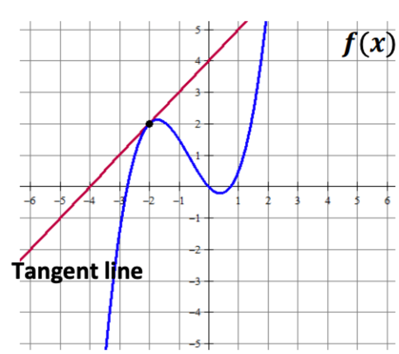
c) (-3, -4), $y = -2$

7) Write the equation of the tangent line in point-slope form. $y - y_1 = m(x - x_1)$

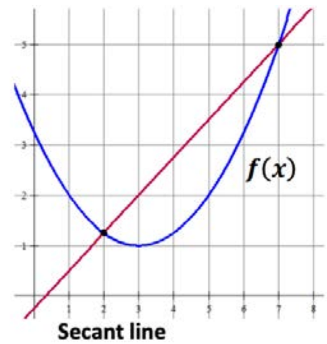
a) The line tangent to $f(x)$ at $x = 1$



b) The line tangent to $f(x)$ at $x = -2$

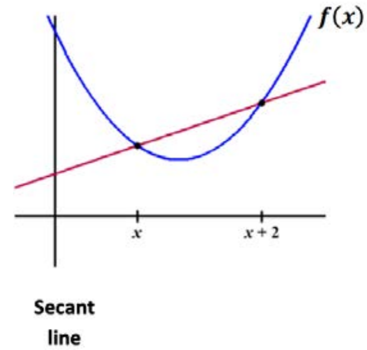


Multiple Choice - Remember slope = $\frac{y_2 - y_1}{x_2 - x_1}$



8) Which choice represents the slope of the secant line shown?

A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$



9) Which choice represents the slope of the secant line shown?

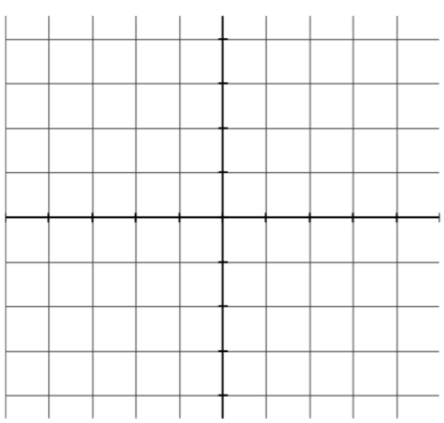
A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$

Topic C: Graphs of Common Functions

Sketch each function as accurately as possible and write the domain and range for each. Some questions will also ask you to find the inverse. You may use a graphing calculator for some of them. If you don't have access to a graphing calculator, I recommend using www.desmos.com. Be accurate with open and closed circles as graphing calculators won't necessarily show these. **You must have all these graphs memorized.**

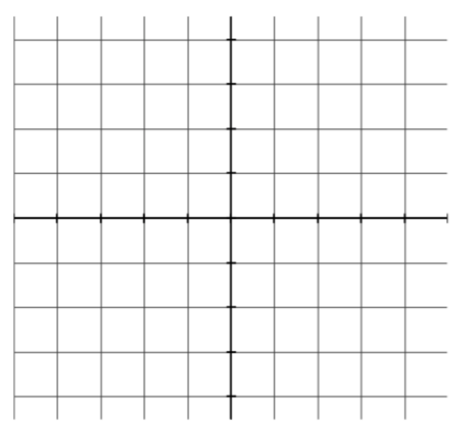
10) $y = x$



Domain: Range:

Inverse:

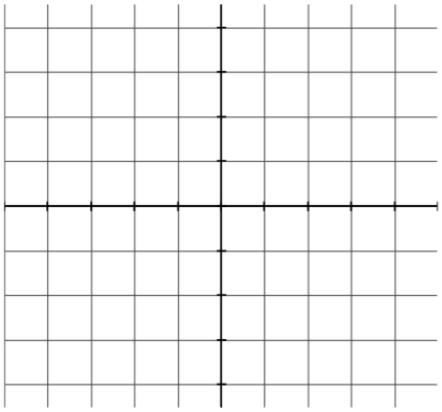
11) $y = x^2$



Domain: Range:

Inverse:

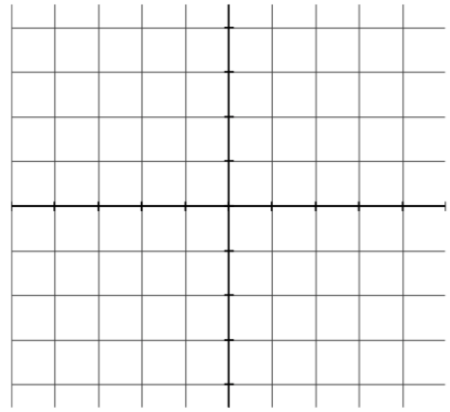
12) $y = x^3$



Domain: Range:

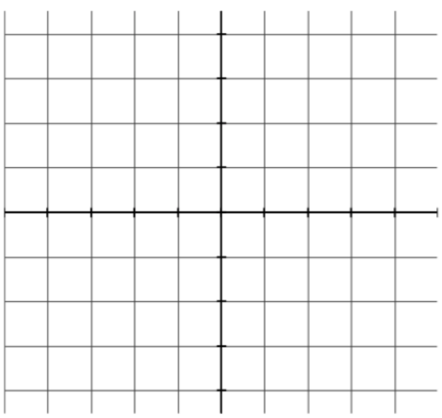
Inverse:

13) $y = \sqrt{x}$



Domain: Range:

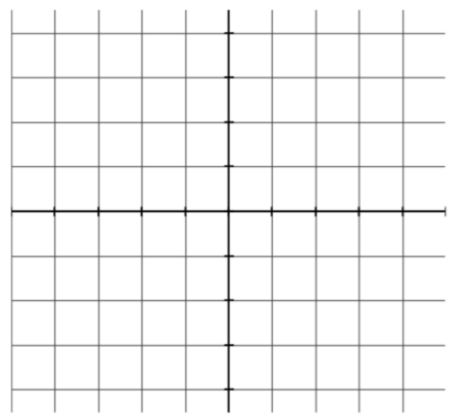
14) $y = |x|$



Domain: Range:

Express as a piecewise function:

15) $y = \frac{|x|}{x}$

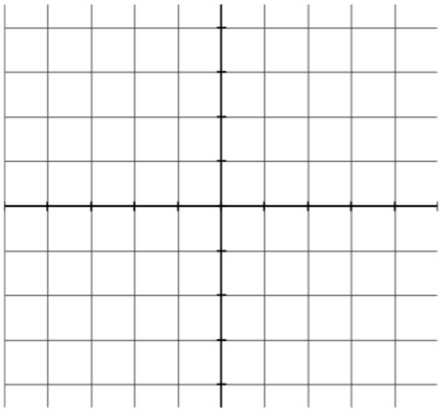


Domain: Range:

Express as a piecewise function:

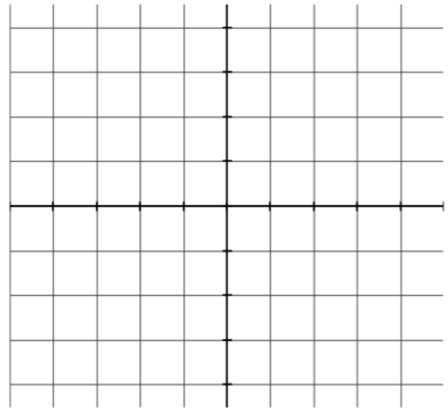
- **This is a very important graph in Calculus.**

16) $y = x^{\frac{1}{3}}$



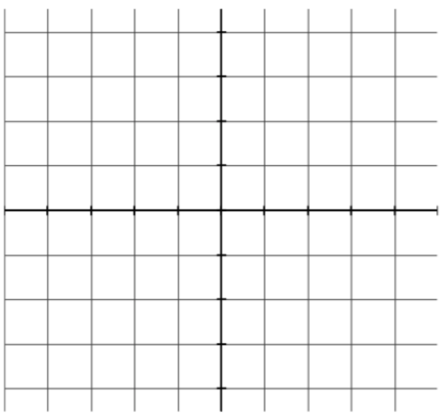
Domain: Range:

17) $y = x^{\frac{2}{3}}$



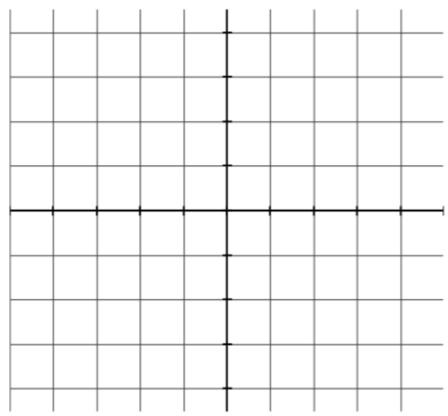
Domain: Range:

18) $y = \frac{1}{x}$



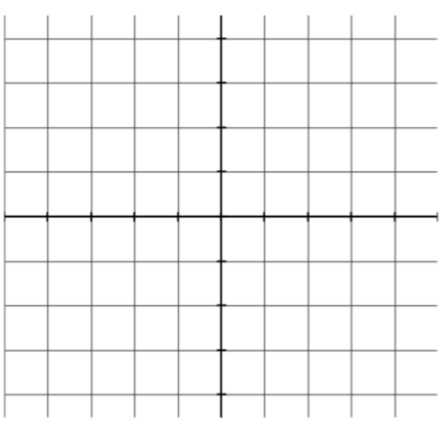
Domain: Range:

19) $y = \frac{1}{x^2}$

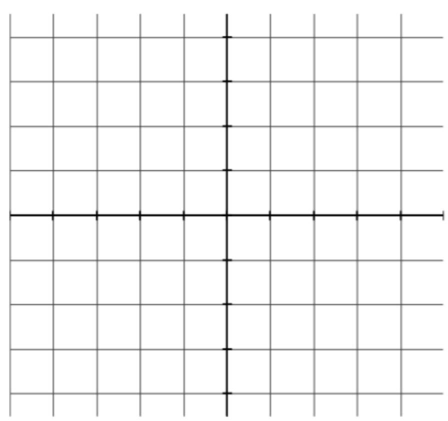


Domain: Range:

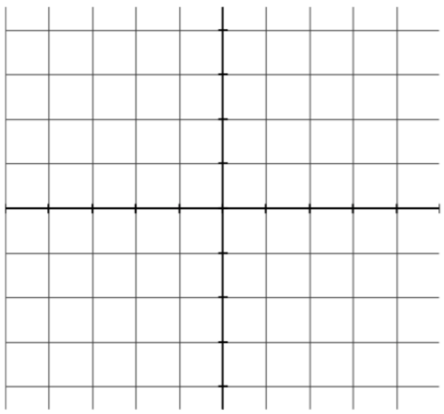
20) $y = 2^x$



21) $y = \sqrt{4 - x^2}$



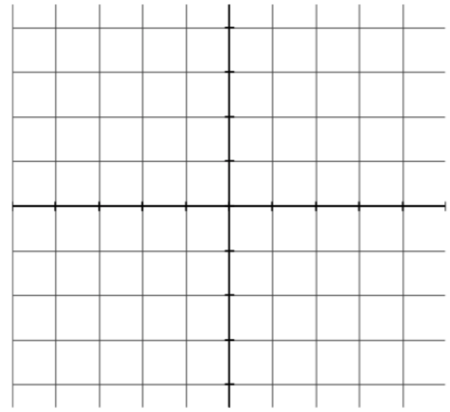
22) $y = e^{x}$



Domain: Range:

Inverse:

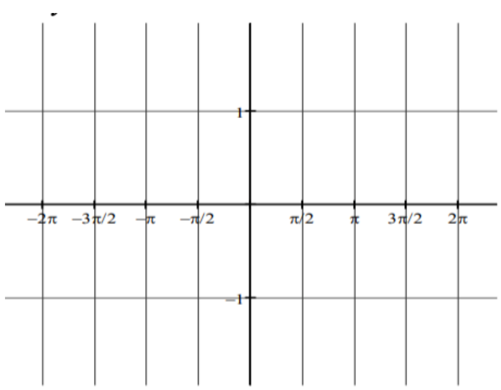
23) $y = ln\, x$



Domain: Range:

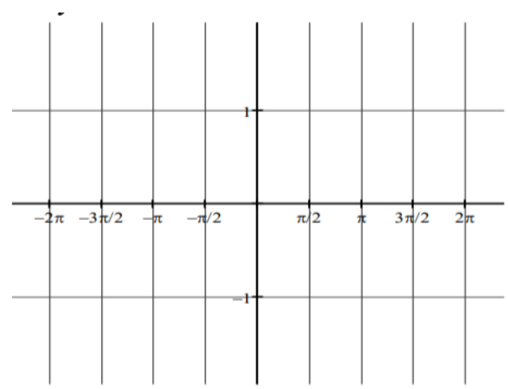
Inverse:

24) $y = sin\, x$



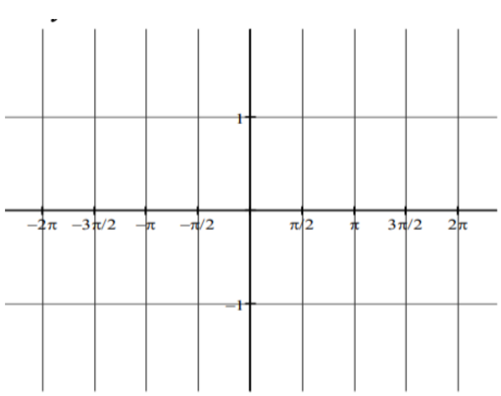
Domain: Range:

25) $y = cos\, x$



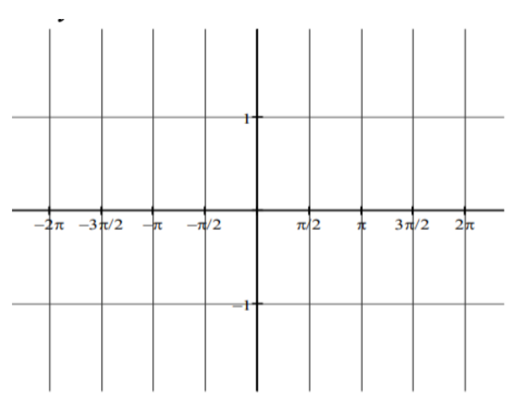
Domain: Range:

26) $y = tan\, x$



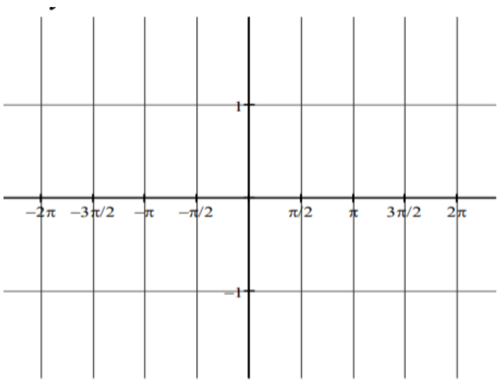
Domain: Range:

27) $y = cot\, x$

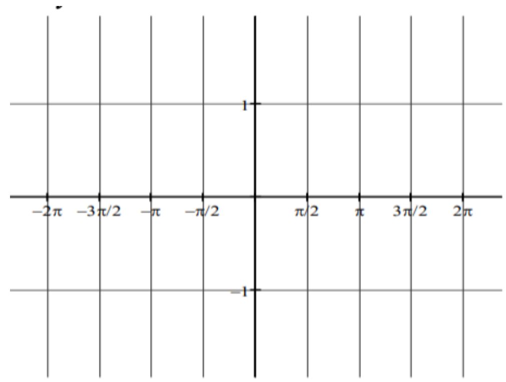


Domain: Range:

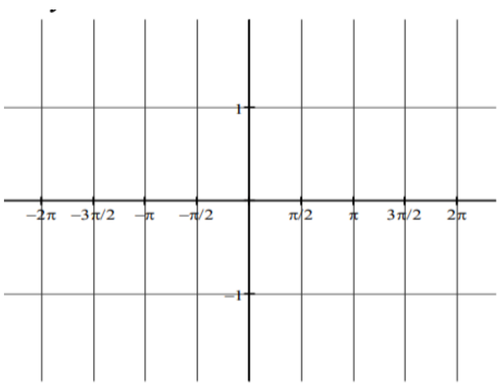
28) $y = sec\,x$



29) $y = csc\,x$

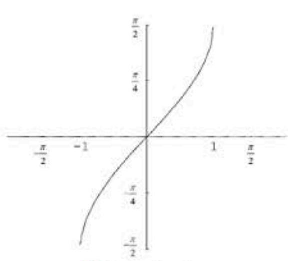
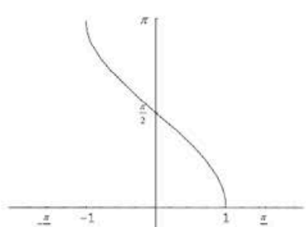
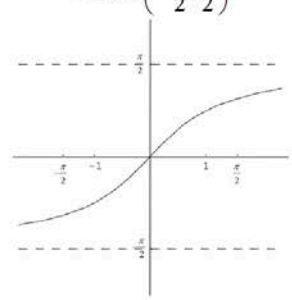


30) $y = \frac{sin\,x}{x}$



It is very important that you know these graphs for AP Calculus!
Please memorize them and their domain and range.

Please also be familiar with the inverse trig functions below.

Domain: $[-1,1]$ Range: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	Domain: $[-1,1]$ Range: $[0,\pi]$	Domain: $(-\infty,\infty)$ Range: $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
		
$f(x)=\sin^{-1}x$ $f(x)=\arcsin x$	$f(x)=\cos^{-1}x$ $f(x)=\arccos x$	$f(x)=\tan^{-1}x$ $f(x)=\arctan x$

Topic D: Function Transformations

31) If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$:

a) $f(x) - 4$

b) $f(x - 4)$

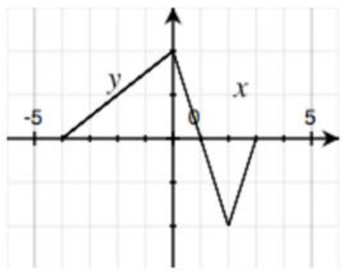
c) $-f(x + 2)$

d) $5f(x) + 3$

e) $f(2x)$

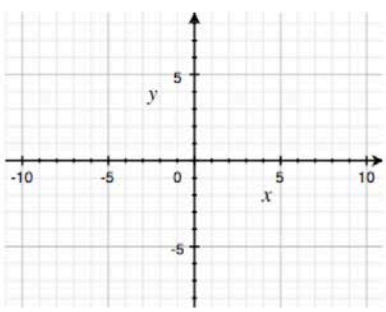
f) $\frac{1}{2}f(-x)$

32) Here is the graph of $f(x)$:

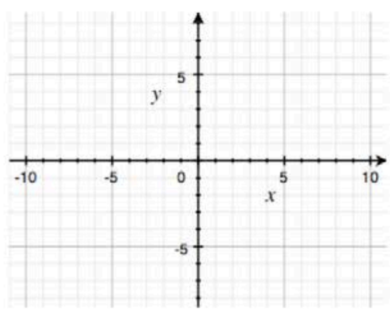


Sketch the following graphs:

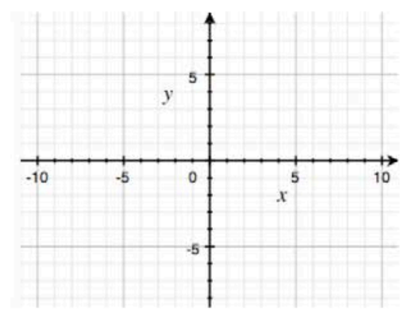
a) $y = 2f(x)$



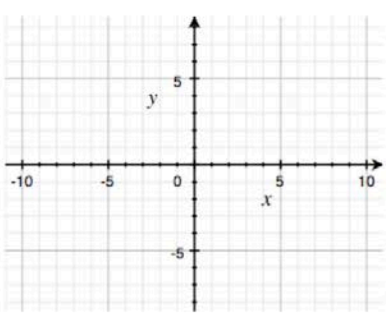
b) $y =- f(x)$



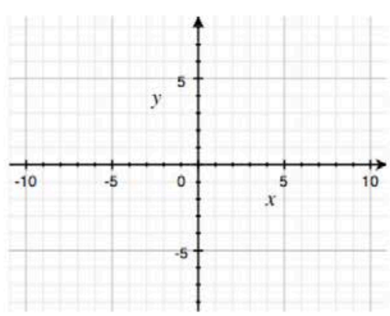
c) $f(x - 1)$



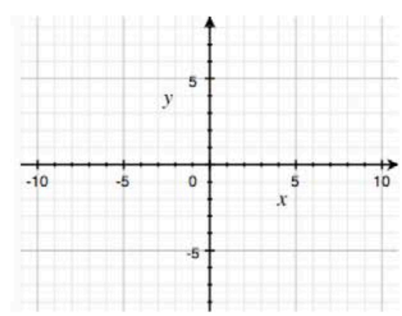
d) $y = f(x) + 2$



e) $y = f(-x)$



f) $y = f(x + 1)$



Topic E: Factoring

Write each expression in factored form. (Remember to look for greatest common factors first)

33) $12a^2b^2 - 3ab$

34) $4x^2 - 9$

35) $x^2 + 2x - 24$

36) $15n^2 - 27n - 6$

37) $x^3 + 8$

38) $x^3 - 8$

39) $x^4 + 11x^2 - 80$

40) $(x - 3)^2(2x + 1)^3 + (x - 3)^3(2x + 1)^2$

41) $2x^{-2} + 8x^{-5}$

42) $6x^{-1/2} + 8x^{1/2} + 2x^{3/2}$

43) $x^2(x^2 + 1)^{-1/2} + (x^2 + 1)^{1/2}$

44) $e^x - xe^{-x}$

45) $2e^{2x} + 5e^x - 12$

46) $e^{x + \Delta x} - e^x$

Topic F: Solving Quadratic and Polynomial Equations

Solve each equation for x over the real number system.

47) $2x^2 - 72 = 0$

48) $12x^2 - 5x = 2$

49) $x^2 + x + \frac{1}{4} = 0$

50) $x^3 - 5x^2 + 5x - 25 = 0$

51) $2x^4 - 15x^3 + 18x^2 = 0$

Topic G: Holes/Asymptotes

For each function, find the equations of both the vertical asymptote(s) and the horizontal asymptote (if it exists) and the location of any holes. *See "Things to Know for Calculus" for a worked out example.*

52) $y = \frac{x-1}{x+5}$

53) $y = \frac{2x+16}{x+8}$

54) $y = \frac{2x^2+6x}{x^2+5x+6}$

55) $y = \frac{x}{x^2-25}$

56) $y = \frac{x^3}{x^2+4}$

57) $y = \frac{10x+20}{x^3-2x^2-4x+8}$

Topic H: Domain and Range

Find the domain of each function. Write in interval notation.
Hint: The domain of a polynomial is all real numbers, fractions cannot lead to division by 0, and there cannot be a negative under a square root.

58) $y = x^3 - x^2 + x$

59) $y = \frac{x^3 - x^2 + x}{x}$

60) $y = \frac{x - 4}{x^2 - 16}$

61) $y = \frac{1}{4x^2 - 4x - 3}$

62) $y = \sqrt{2x - 9}$

63) $y = \frac{\sqrt{2x + 14}}{x^2 - 49}$

Identify the domain and range of each function. Describe the transformation from the parent function.

64) $g(x) = |x - 2| + 1$ Domain: Range:

Transformation(s):

65) $h(x) = - |x - 2|$ Domain: Range:

Transformation(s):

66) $f(x) = log(x - 10)$ Domain: Range:

Transformation(s):

67) $g(x) = \frac{2}{x-1} + 4$ Domain: Range:

Transformation(s):

Topic I: Complex Fractions

Eliminate the complex fractions. *See "Things to Know for Calculus" for a worked out example.*

68) $\dfrac{2a - \frac{1}{8a}}{4 + \frac{1}{a}}$

69) $\dfrac{x - \frac{1}{x}}{x + \frac{1}{x}}$

70) $\dfrac{1}{2 - \frac{1}{m-4}}$

71) $\dfrac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}}$

72) $\dfrac{5 - \frac{x}{x-1}}{2 - \frac{x}{x+1}}$

73) $\dfrac{\frac{1}{x+h} - \frac{1}{x}}{h}$

Topic J: Rationalization

Rationalize each expression by multiplying by the conjugate. (The conjugate of $a + b$ is $a - b$)

See "Things to Know for Calculus" for a worked out example.

74) $\dfrac{2}{5+\sqrt{3x}}$

75) $\dfrac{x}{x-\sqrt{x+3}}$

76) $\dfrac{3-\sqrt{x}}{x-9}$

77) $\dfrac{\sqrt{x+7}\ -2}{-x+3}$

78) $\dfrac{4-\sqrt{18-x}}{x-2}$

79) $\dfrac{\sqrt{x-5}-\sqrt{5}}{x-10}$

Topic K: Exponential and Logarithmic Properties

80) Rewrite each expression using the properties of exponents. There should be only positive exponents.

a) $(x^2)(x^3) =$ b) $\frac{x^2}{x^3} =$ c) $(3^x)^3 =$ d) $(\frac{1}{x})^{-3} =$

81) Rewrite each expression using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

a) $\sqrt{x + 1}$ b) $\frac{1}{\sqrt{x+1}}$ c) $\sqrt[5]{x^3} + \sqrt[5]{2x}$

82) Use the properties of exponents to write each expression in radical form and with positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

a) $(x + 4)^{-\frac{1}{2}}$ b) $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$ c) $3x^{\frac{1}{3}}$

83) Expand the following using the properties of logs.

a) $ln\frac{10}{9}$ b) $ln(3x + 2)^4$ c) $ln(\frac{xy}{z})$

84) Simplify each expression.

a) $log_2 \frac{1}{4}$ b) $log_8 4$ c) $ln\frac{1}{\sqrt[3]{e^2}}$

d) $5^{log_5 40}$ e) e^{lnx} f) $log_{12} 2 + log_{12} 9 + log_{12} 8$

Topic L: Exponential Functions and Logarithms

Solve each exponential and logarithmic equation. Remember: $e^0 = 1$ and $ln(1) = 0$

86) $e^x + 1 = 2$

87) $e^{2x} = 1$

88) $e^x + xe^x = 0$

89) $e^{2x} - e^x = 0 \; ln\, x = 0$

90) $3^{x-2} = 18$

91) $ln\, x = 0$

92) $3 - ln\, x = 3$

93) $ln(3x) = 0$

94) $log_5(3x - 8) = 2$

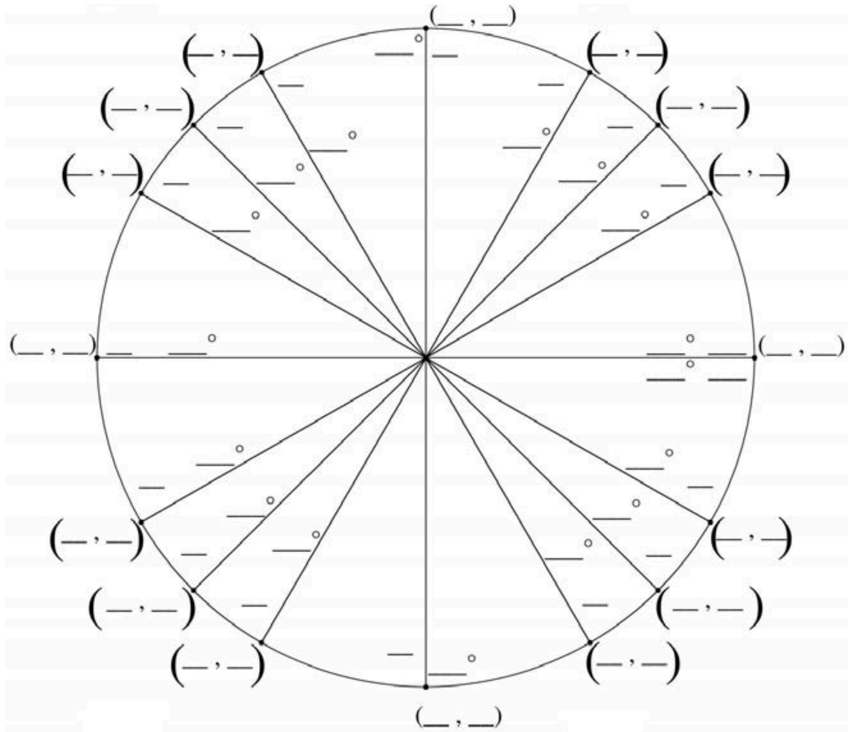
95) $log(x - 3) + log\, 5 = 2$

96) $log_5(x + 3) - log_5 x = 2$

97) $lnx^3 - lnx^2 = \frac{1}{2}$

Topic M: The Unit Circle and Evaluating Trigonometric Functions

98) Fill in the blank unit circle with the degrees, radians, and coordinate points. **This must be memorized!**



99) You need to know the basic trigonometric functions in **radians.** Note that we will never use degrees in calculus. Use the unit circle to find each of the following.

a) $sin\ \frac{\pi}{6}$

b) $cos\ \frac{\pi}{4}$

c) $sin\ 2\pi$

d) $tan\ \pi$

e) $sec\ \frac{\pi}{2}$

f) $cos\ \frac{\pi}{6}$

g) $sin\ \frac{\pi}{3}$

h) $sin\ \frac{3\pi}{2}$

i) $tan\ \frac{\pi}{4}$

j) $csc\ \frac{\pi}{2}$

k) $sin\ \pi$

l) $cos\ \frac{\pi}{3}$

m) Find x where $0 \leq x \leq 2\pi$

$sin\ x = \frac{1}{2}$

n) Find x where $0 \leq x \leq 2\pi$

$tan\ x = 0$

o) Find x where $0 \leq x \leq 2\pi$

$cos\ x = -1$

Topic N: Trigonometric Identities & Angles

100) Fill-in the blanks using **Reciprocal Identities**

a) $sin\,\theta\; =$ b) $cos\,\theta\; =$ c) $tan\,\theta\; =$

d) csc θ = e) sec θ = f) cot θ =

101) Fill-in the blanks using **Quotient Identities**

a) tan θ = b) cot θ =

102) Fill in the blanks using the **Cofunction Identities.**

a) $sin\left(\frac{\pi}{2} - \theta\right)$ =______________ b) $cot\left(\frac{\pi}{2} - \theta\right) =$ __________

c) $cos\left(\frac{\pi}{2} - \theta\right) =$ ___________ d) $sec\left(\frac{\pi}{2} - \theta\right) =$ __________

103) What are the three **Pythagorean Identities**?

__________________________ __________________________ __________________________

104) Fill in the blanks using the Double Angle Identities.

a) $sin\;2\theta\; =$ b) $cos\;2\theta\; =$ c) $tan\;2\theta\; =$

105) If point P is on the terminal side of θ, find all 6 trigonometric functions of θ.

P (-2, 4)

106) State the quadrant for which each of the following is true.

a) $sin\,\theta\; >\; 0$ and $cos\,\theta\; <\; 0$ b) $tan\,\theta\; >\; 0$ and $sec\,\theta\; <-\; 0$

Topic O: Solving Trigonometric Equations

Solve the following trigonometric equations where $0 \leq x \leq 2\pi$.

107) $sin\, x = \frac{1}{2}$

108) $cos\, x = -1$

109) $cos\, x = \frac{\sqrt{3}}{2}$

110) $2sin\, x = -1$

111) $cos\, x = \frac{\sqrt{2}}{2}$

112) $cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

113) $tan\, x = 0$

114) $sin\,(2x) = 1$

115) $sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

Resource Taken From: https://calculus.flippedmath.com

Things to Know for Calculus

TRIGONOMETRY

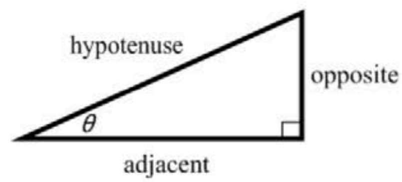
Trig Functions

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ | $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ | $\tan\theta = \frac{\text{opp}}{\text{adj}}$

Reciprocal Functions

$\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}}$ | $\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}}$ | $\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adj}}{\text{opp}}$

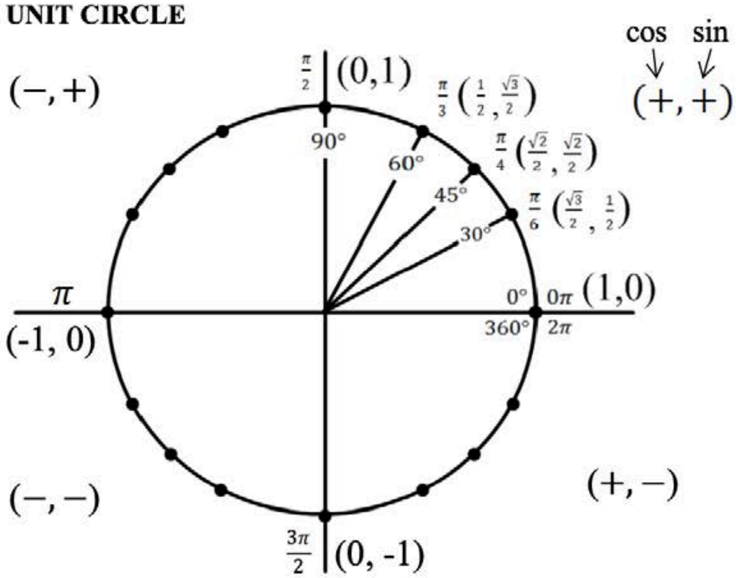
SOH-CAH-TOA



TEST ONLY USES RADIANS!

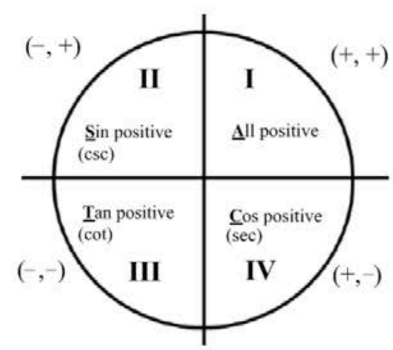
Must know trig values of special angles $0\pi, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ using Unit Circle or Special Right Triangles.

UNIT CIRCLE



To help remember the signs in each quadrant

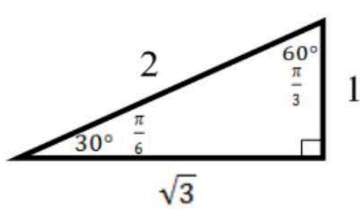
All **S**tudents **T**ake **C**alculus



SPECIAL RIGHT TRIANGLES

$30° - 60° - 90°$ Triangles

Which are $\frac{\pi}{6} - \frac{\pi}{3} - \frac{\pi}{2}$ Triangles

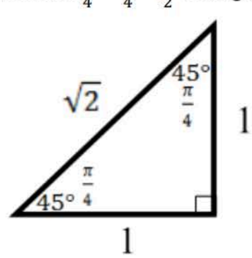


Find $\tan\left(\frac{\pi}{6}\right)$

$\tan\left(\frac{\pi}{6}\right) = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}}$ simplify to $\frac{1}{\sqrt{3}} \frac{\cdot\sqrt{3}}{\cdot\sqrt{3}} = \frac{\sqrt{3}}{3}$

$45° - 45° - 90°$ Triangles

Which are $\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}$ Triangles

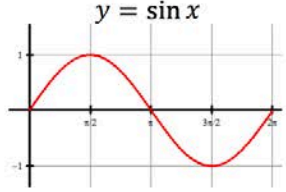


Find $\sin\left(\frac{\pi}{4}\right)$

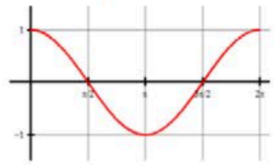
$\sin\left(\frac{\pi}{4}\right) = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$ simplify to $\frac{1}{\sqrt{2}} \frac{\cdot\sqrt{2}}{\cdot\sqrt{2}} = \frac{\sqrt{2}}{2}$

Graphs of trig functions

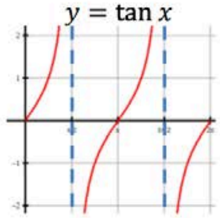
$y = \sin x$



$y = \cos x$



$y = \tan x$



Inverse Trig Function

$\sin^{-1}\theta$ is the same as $\arcsin\theta$

$\sin^{-1}\theta = \left(\frac{\sqrt{3}}{2}\right)$ means what angle has a sine value of $\frac{\sqrt{3}}{2}$

that means $\theta = \frac{\pi}{3} \pm 2\pi n$ *or* $\frac{2\pi}{3} \pm 2\pi n$

Since θ has infinite answers then it isn't a function. Bummer. To make it a function we define inverses like:

sin/csc and tan/cot use quadrant I and IV for inverses
cos/sec use quadrant I and II for inverses

So… $\theta = \frac{\pi}{3}$ because it is in the first quadrant

Trig Identities

There are a bunch, but you really only need to know Pythagorean Identity. $\mathbf{\sin^2 x + \cos^2 x = 1}$

Subtract $\sin^2 x$ to get $\cos^2 x = 1 - \sin^2 x$ or subtract $\cos^2 x$ to get $\sin^2 x = 1 - \cos^2 x$

Divide by $\sin^2 x$ to get $1 + \cot^2 x = \csc^2 x$ or divide by $\cos^2 x$ to get $\tan^2 x + 1 = \sec^2 x$

GEOMETRY

FORMULAS

AREA

Triangle $= \frac{1}{2}bh$

Circle $= \pi r^2$

Trapezoid $= \frac{1}{2}(b_1 + b_2)h$

CIRCUMFERENCE

Circle $= 2\pi r$

SURFACE AREA

Sphere $= 4\pi r^2$

LATERAL AREA

Cylinder $= 2\pi rh$

VOLUME

Sphere $= \frac{4}{3}\pi r^3$

Cylinder $= \pi r^2 h$

Cone $= \frac{1}{3}\pi r^2 h$

Prism $= Bh$

Pyramid $= \frac{1}{3}Bh$

B is the area of the base

DISTANCE FORMULA

The distance between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

ALGEBRA

Linear Functions

Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept Form
(slope-intercept Form)

$$y = mx + b$$

Point Slope Form

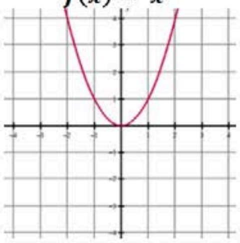
$$y - y_1 = m(x - x_1)$$

Parallel Lines
Have the same slope

Perpendicular Lines
Have the opposite reciprocal slopes

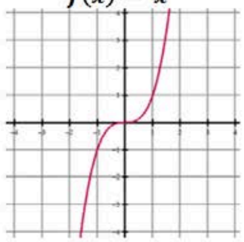
Functions

Quadratic Function
$f(x) = x^2$



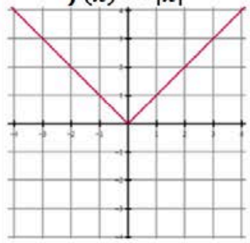
$y = a(x-h)^2 + k$

Cubic Function
$f(x) = x^3$



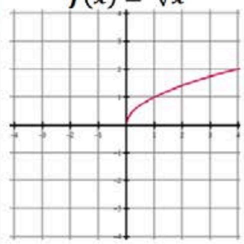
$y = a(x-h)^3 + k$

Absolute Value
$f(x) = |x|$



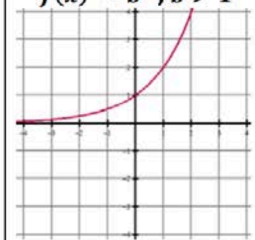
$y = a|x-h| + k$

Square Root Function
$f(x) = \sqrt{x}$



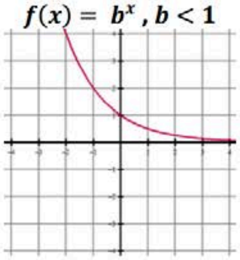
$y = a\sqrt{x-h} + k$

Exponential Function
$f(x) = b^x, b > 1$



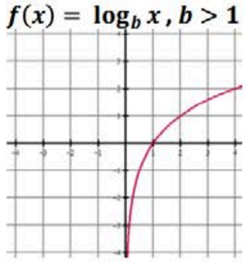
$y = a \cdot b^{(x-h)} + k$

Exponential Function
$f(x) = b^x, b < 1$



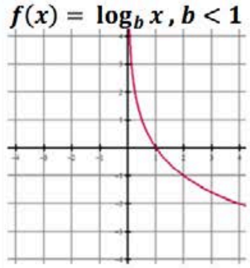
$y = a \cdot b^{(x-h)} + k$

Logarithmic Function
$f(x) = \log_b x, b > 1$



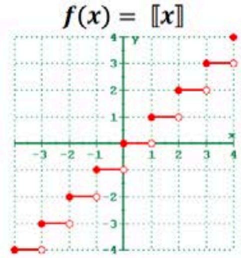
$y = a\log_b(x-h) + k$

Logarithmic Function
$f(x) = \log_b x, b < 1$



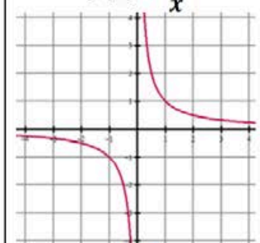
$y = a\log_b(x-h) + k$

Greatest Integer
$f(x) = [\![x]\!]$



$y = a[\![x-h]\!] + k$

Rational Function
$f(x) = \frac{1}{x}$



$y = \frac{a}{x-h} + k$

Translations

All functions move the same way!

Given the parent function $y = x^2$

Move up 4	Move down 3	Move left 2	Move right 1	Move left 2 and down 3
$y = x^2 + 4$	$y = x^2 - 3$	$y = (x+2)^2$	$y = (x-1)^2$	$y = (x+2)^2 - 3$

To flip (reflect) the function vertically $y = -x^2$
To flip (reflect) the function horizontally $y = (-x)^2$

So $f(x) = -\sqrt{x-3} + 1$ is a square root function reflected vertically, shifted right 3 and up 1

Notation

Notice open parenthesis ( ) versus closed [ ]

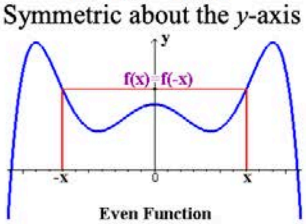
Inequality		Interval
$-3 < x \le 5$	$\longleftrightarrow$	$(-3,5]$
$-3 \le x \le 5$	$\longleftrightarrow$	$[-3,5]$
$-3 < x < 5$	$\longleftrightarrow$	$(-3,5)$
$-3 \le x < 5$	$\longleftrightarrow$	$[-3,5)$

Infinity is always open parenthesis

Inequality		Interval
$x < 3$	$\longleftrightarrow$	$(-\infty, 3)$
$x \le 3$ or $x > 5$	$\longleftrightarrow$	$(-\infty, 3](5, \infty)$
$x \ne 3$	$\longleftrightarrow$	$(-\infty, 3)(3, \infty)$
all Real numbers	$\longleftrightarrow$	$(-\infty, \infty)$

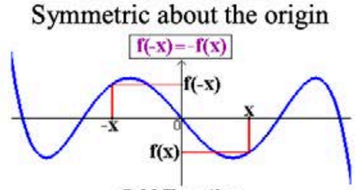
Even and Odd Functions

EVEN
$f(-x) = f(x)$
Symmetric about the y-axis



Even Function

ODD
$f(-x) = -f(x)$
Symmetric about the origin

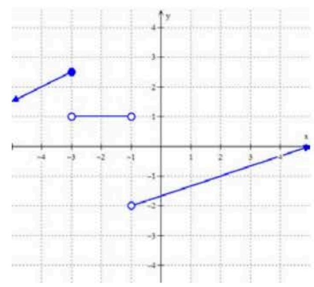


Odd Function

Domain and Range

Domain = all possible x values

Range = all possible y values

Algebraically You can't divide by zero You can't square root a negative	Graphically Just look at it
$y=\sqrt{2x+5}$ D: $[-\frac{5}{2},\infty)$ $y=\frac{x^2-1}{x^2+7x+12}$ D: $(-\infty,-4)(-4,-3)(-3,\infty)$	 D: $(-\infty,-1)(-1,5]$ R: $(-\infty,2.5]$

Finding zeros

Must be able to factor and use the quadratic formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Special products

Sum of cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$

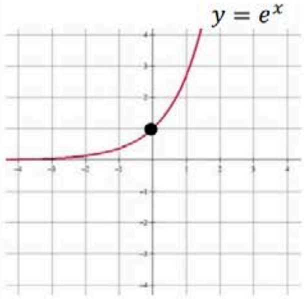
Difference of cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$

Exponential and Logarithmic Properties

The exponential function b^x of base b is one-to-one which means it has an inverse which is called the logaritmic function of base b or logarithm of base b which is denoted $\log_b x$ which reads "the logarithm of base b of x" or "log base b of x". So…

BASE e

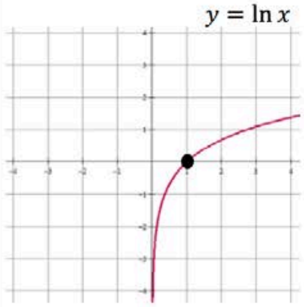
$y=e^x$



KNOW!
$e^0=1$

Horizontal asymptote at $y=0$

$y=\ln x$



KNOW!
$\ln 1=0$

Vertical asymptote at $x=0$

$y=\log_b x \longleftrightarrow x=b^y$

Exponential		Logarithmic
$b^x b^y=b^{x+y}$	Product Rule	$\log_b xy=\log_b x+\log_b y$
$\frac{b^x}{b^y}=b^{x-y}$	Quotient Rule	$\log_b\left(\frac{x}{y}\right)=\log_b x-\log_b y$
$(b^x)^y=b^{xy}$	Power Rule	$\log_b x^y=y\log_b x$
$b^{-x}=\frac{1}{b^x}$		$\log_b\left(\frac{1}{x}\right)=-\log_b x$
$b^0=1$		$\log_b 1=0$
$b^1=b$		$\log_b b=1$
	Change of Base	$\log_b x=\frac{\log_c x}{\log_c b}$
	Natural Log	$\log_e x=\ln e$
	Common Log	$\log_{10} x=\log x$

Worked Out Examples

Rational Functions: Finding Holes and Asymptotes

A rational function is the quotient of two polynomial functions. Ex: $f(x) = \frac{x-2}{x^2+3x+1}$

Holes occur when there is a common factor between the numerator and denominator.

Steps:
1) Factor the numerator and denominator. Look for any common factors.
2) Set the common factor equal to 0 and solve for x. This is the x coordinate of the hole.
3) Substitute the x value into the remaining function to find the y coordinate of the hole.

An asymptote is a line representing a value that the function approaches but never reaches. There are three types of asymptotes: Vertical, Horizontal, and Slant.

A **vertical asymptote** occurs when the denominator is equal to 0 after all common factors have been removed.

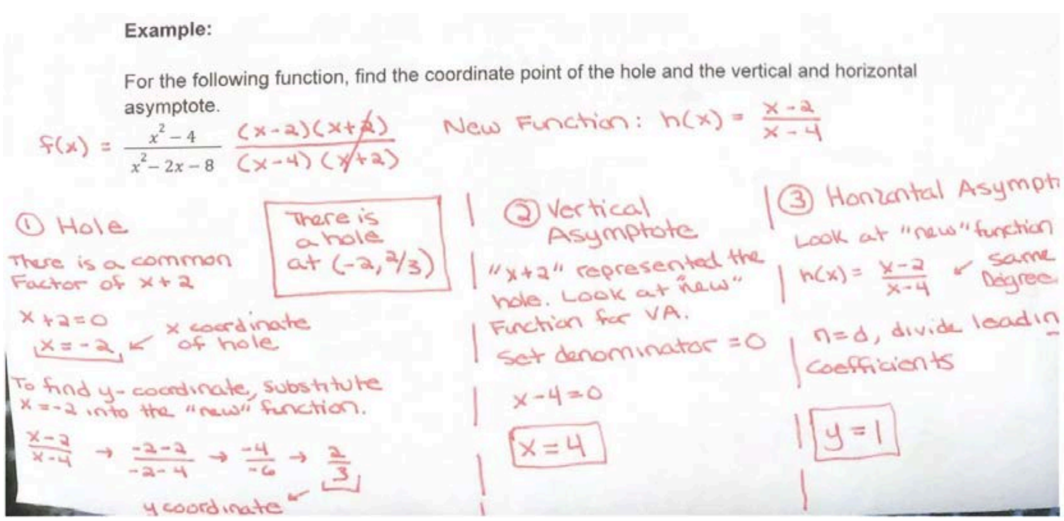
A **horizontal asymptote** is found by comparing the degree (highest exponent) of the numerator (n) with the degree of the denominator (d). Follow these rules:

- If $n < d$, the horizontal asymptote is $y = 0$
- If $n = d$, the horizontal asymptote is $y = \frac{\text{Leading coefficient of numerator}}{\text{Leading coefficient of denominator}}$
- If $n > d$, there is no horizontal asymptote

Example:

For the following function, find the coordinate point of the hole and the vertical and horizontal asymptote.

$\frac{x^2-4}{x^2-2x-8}$

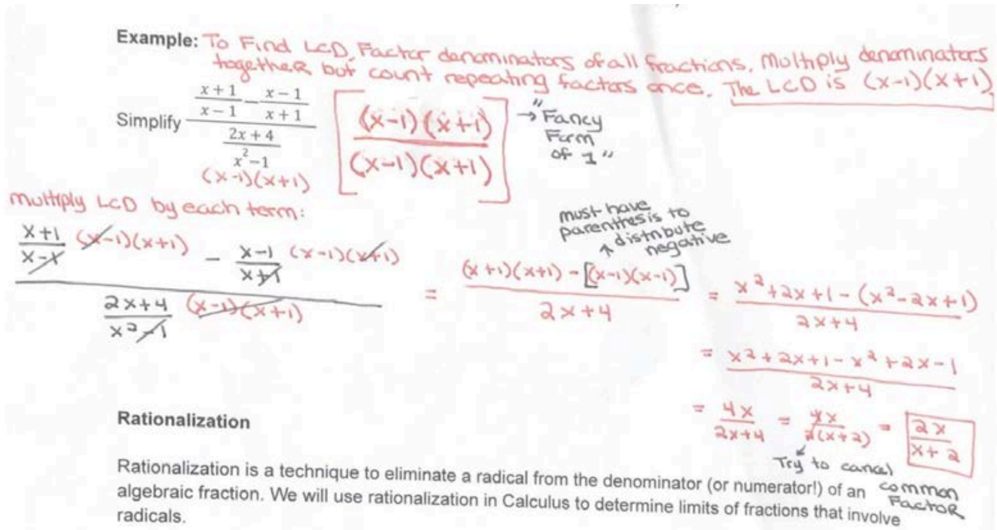


Complex Fractions

Complex Fractions have numerators and denominators that contain fractions. To simplify complex fractions, multiply the numerator and denominator by the least common denominator. Remember, the LCD is typically the denominators multiplied together (with no repeats).

Example:

Simplify $\dfrac{\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}}{\dfrac{2x+4}{x^2-1}}$



Rationalization

Rationalization is a technique to eliminate a radical from the denominator (or numerator!) of an algebraic fraction. We will use rationalization in Calculus to determine limits of fractions that involve radicals.

The method involves multiplying by the **conjugate**. A binomial conjugate is when the sign differs in the middle of the two terms (a + b and a - b are conjugates). When conjugates are multiplied by FOIL, the middle terms cancel: $(a + b)(a - b) = a^2 - b^2$.

Example:

Rationalize the numerator: $\dfrac{3-\sqrt{5x-1}}{x-2}$