

Intro. to BC Calculus

Summer Assignment

Name _____

Happy Summer Intro. to BC Calculus students-to-be;

This summer packet includes practice problems of skills and concepts you have been introduced to and have worked with in your previous math courses. The purpose of this summer packet is to refresh your memory about concepts, which by the end of the summer may seem a bit distant, so that in September we can “hit the ground running”. I recommend that you start your work mid-August as it must be completed and turned in at the beginning of our first day of class. I will not accept this packet late or if it is incomplete, so please manage your time wisely and plan accordingly. You will be given the entire first class to ask questions for clarification and for review of what is in this packet. During our second class you will be tested on the material you have revisited in this summer packet and only this material. Although I encourage you to use your calculator to check your work as you practice, please note that the Test will be taken without the aid of a calculator. This Test will represent the first major grade of the first quarter. There are no “make-up’s” for this Test. If you are not completely solid with these foundational skills it is an indicator that you may have difficulty meeting with success in this rigorous course. If you need to contact me, feel free to email me at cflanders@mvyps.org. Enjoy your summer and I’ll be looking forward to meeting you in September.

Mrs. Flanders

MVRHS

Math Department Chair

Topics for Review:

- * The Linear Function
- * Factoring
- * The Quadratic Function
- * Fundamental Theorem of Algebra
- * Polynomial Functions of Higher Degree
- * Simplifying Radicals
- * Rationalizing
- * Simplifying Complex Fractions
- * Composite and Inverse Functions
- * Parent Functions and Transformations

The Linear Function: $f(x) = x$

1. Given a line with $m = -\frac{8}{3}$:

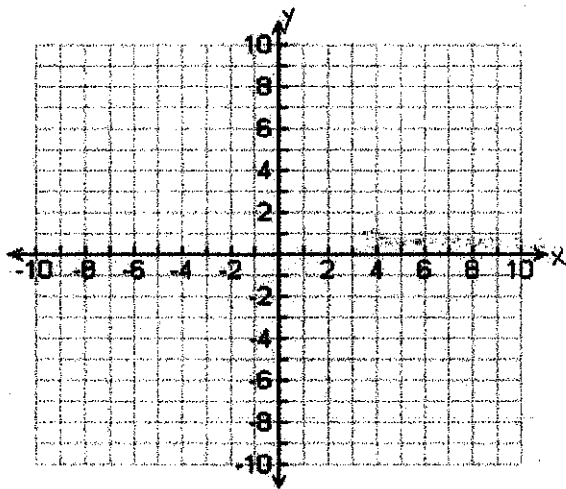
- a) Is this line increasing or decreasing? Explain.
- b) Quickly or slowly? Explain.

2. Given a line that passes through the points (5, 1) and (-6, -3) :

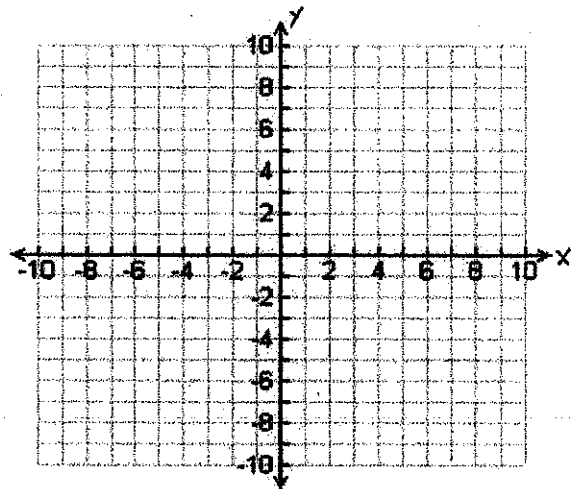
- a) Is this line increasing or decreasing? Explain.
- b) Quickly or slowly? Explain.

Graph the following linear equations:

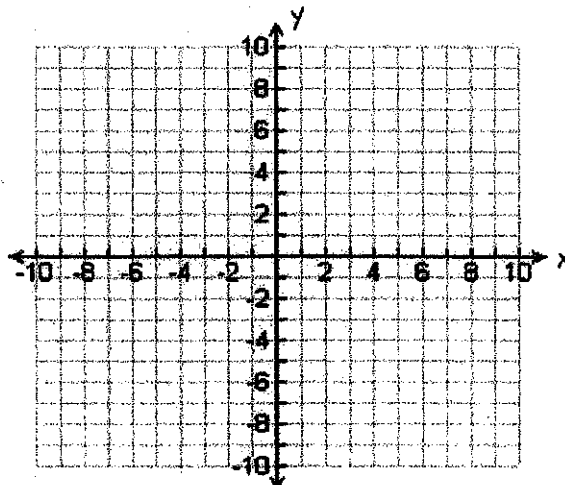
3. $y = -3x + 1$



4. $-2x + 4y = 12$

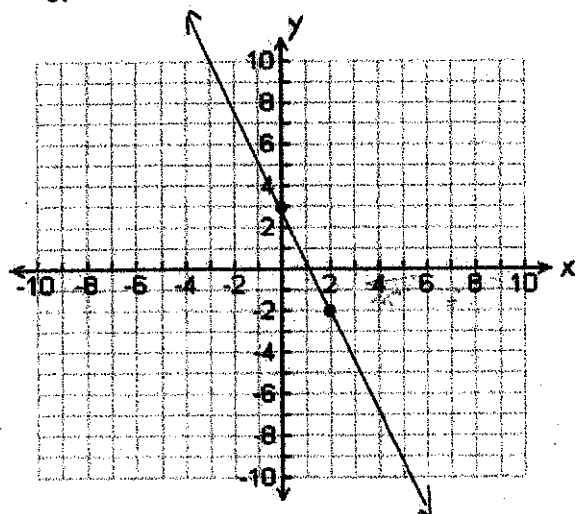


5. $y - 1 = \frac{2}{3}(x + 4)$

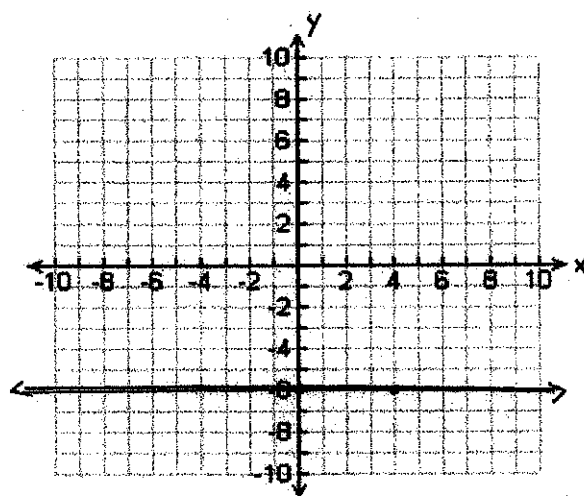


Given the following graphs, write an equation for the line:

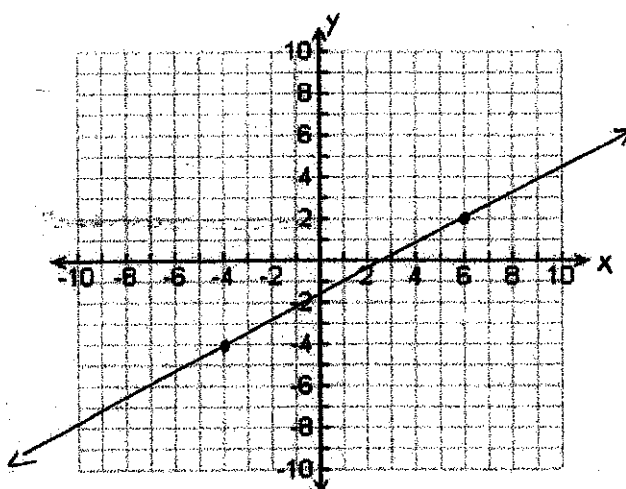
6.



7.



8.



9. Write an equation for a line that is parallel to $y = \frac{3}{4}x + 1$ and passes through the point (2, 7)

10. Write an equation for a line that is perpendicular to $2x - 3y = 8$ and passes through the point (-1, 4)

Solve the following systems of linear equations:

11.
$$\begin{cases} y = 2x + 5 \\ y = -6x - 3 \end{cases}$$

12.
$$\begin{cases} 2x - 4y = 24 \\ -3x + y = -11 \end{cases}$$

13.
$$\begin{cases} x = 7 + 3y \\ -2y + 8x = 56 \end{cases}$$

Factoring

Factor the following completely:

1. $15x^2 + 3x$

2. $x^2 - 25$

3. $4 - 9x^2$

4. $x^3 + 27$

5. $125 - x^3$

6. $8x^3 + 1$

7. $x^2 + 16x + 64$

8. $x^2 - 6x + 9$

9. $x^2 + x - 20$

10. $x^2 - 6x + 8$

11. $x^2 + 11x + 24$

12. $3x^2 + 22x + 7$

13. $4x^2 + 11x + 6$

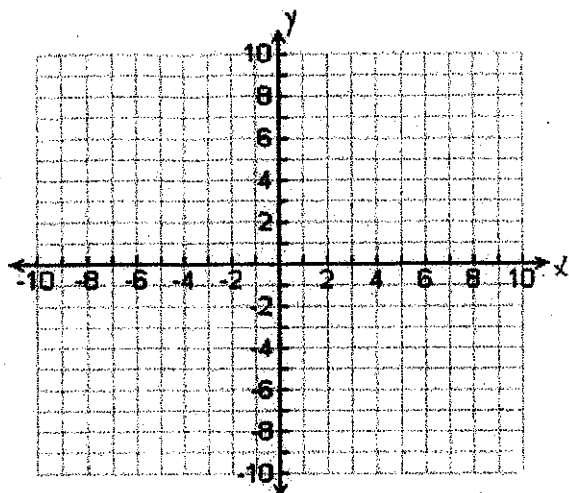
14. $ax + 3bx - ay - 3by$

15. $21x^3 - 84x^2 + 15x - 60$

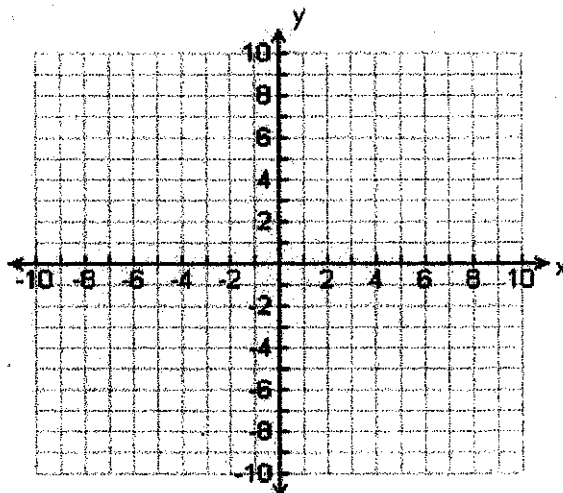
The Quadratic Function: $f(x) = x^2$

- * Graph the following quadratic equations: (mark at least 3 points)
- * State the coordinates of the vertex:
- * State whether the vertex is a maximum or a minimum:
- * Write an equation for the line of symmetry:
- * State the Domain and Range:

1. $y = -x^2 + 2x + 3$

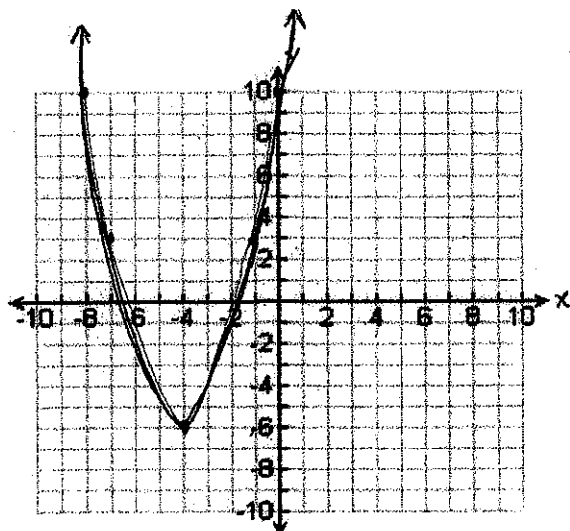


2. $y = (x - 3)^2 - 4$

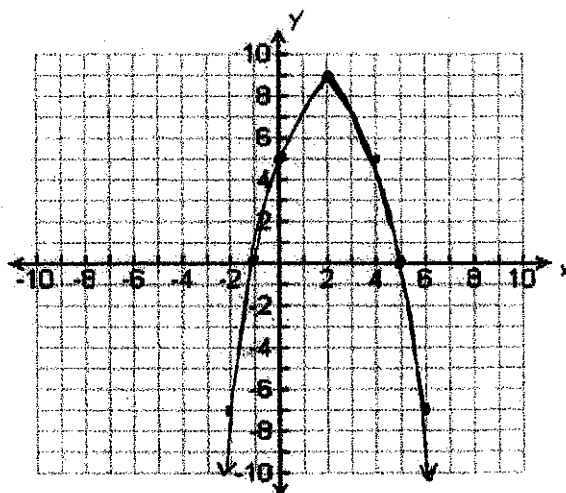


- * Write an equation for the graphed quadratic equations:
- * State the coordinates of the vertex:
- * State whether the vertex is a maximum or a minimum:
- * Write an equation for the line of symmetry:
- * State the Domain and Range:

3.



4.



Solve by taking the square root:

5. $x^2 = 81$

6. $3x^2 - 4 = 44$

Solve by factoring:

7. $x^2 + 2x = 15$

8. $2x^2 - 10x - 12 = 0$

Solve by using the quadratic formula:

9. $x^2 + 3x = 8$

Solve by completing the square:

10. $x^2 - 6x - 10 = 0$

The Fundamental Theorem of Algebra

* State the number of possible zeros;

* List the possible rational roots:

1. $f(x) = 5x^4 - 36x^2 - 8$

2. $f(x) = 4x^5 + 3x^4 + 140x^3 + 28x^2 + 45x + 9$

Divide using long division:

3. $\frac{x^4 + x^3 + 7x^2 - 6x + 8}{x^2 + 2x + 8}$

4. $(x^3 + 5x^2 - 32x - 7) \div (x - 4)$

5. $5k - 4 \overline{) 50k^3 + 10k^2 - 35k - 7}$

Divide using synthetic division:

6. $\frac{5x^3 - 16x^2 + 16x - 8}{x - 2}$

7. $(k^2 - 7k + 10) \div (k - 1)$

8. $x + 6 \overline{) x^2 + 5x + 3}$

Polynomial Functions of Higher Degree

* Determine the real zeros and state the multiplicity of any repeated zeros:

* State whether each zero crosses the x-axis or not:

1. $f(x) = x^4 + x^3$

2. $f(x) = -x^3 + 6x^2 - 12x + 8$

3. $f(x) = 2x^3 - 3x^2$

4. $f(x) = -x^4 + x^3 - 4x^2 + 4x$

* State the number of possible extrema:

* Describe the end behavior:

5. $f(x) = -x^5 + 2x^3 - x + 1$

6. $f(x) = 2x^2 - 4x - 3$

7. $f(x) = -x^4 - 9x^2 - 24x - 20$

8. $f(x) = x^3 - 2x^2 - x + 1$

Simplifying Radicals / Simplifying Powers with Rational Exponents

Simplify the following:

1. $\sqrt{24}$

2. $8^{\frac{2}{3}}$

3. $3\sqrt{32}$

4. $16^{\frac{-3}{4}}$

5. $-2\sqrt{192}$

6. $-125^{\frac{2}{3}}$

7. $\frac{4}{\sqrt{50}}$

8. $\left(\frac{1}{27}\right)^{\frac{-1}{3}}$

9. $\frac{2\sqrt{3}}{\sqrt{6}}$

10. $\left(\frac{1}{32}\right)^{\frac{2}{5}}$

Rationalizing

Rationalize the Denominator:

1. $\frac{2y}{\sqrt{18y^3}}$

2. $\frac{\sqrt{25}}{\sqrt{3x}}$

3. $\frac{2}{5 + \sqrt{3x}}$

4. $\frac{x}{x - \sqrt{x+3}}$

Rationalize the Numerator:

5. $\frac{3 - \sqrt{x}}{x - 9}$

6. $\frac{\sqrt{x+7} - 2}{x + 3}$

7. $\frac{4 - \sqrt{18-x}}{x - 2}$

8. $\frac{\sqrt{5+x} - \sqrt{5}}{x}$

Simplifying Complex Fractions

Simplify the following:

1.
$$\frac{2a - \frac{1}{8a}}{4 + \frac{1}{a}}$$

2.
$$\frac{1}{2 - \frac{1}{m-4}}$$

3.
$$\frac{1}{1 - \frac{1}{x-1}}$$

4.
$$\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}}$$

5.
$$\frac{5 - \frac{x}{x-1}}{2 - \frac{x}{x+1}}$$

6.
$$\frac{x^{-3}y - 2x^2y^{-1}}{x^{-1} + y^{-1}}$$

Composite and Inverse Functions

The Composition of Functions

The composition of the function f with g is denoted by $f \circ g$ and is defined by the equation

$$(f \circ g)(x) = f(g(x))$$

The domain of the composite function $f \circ g$ is the set of all x such that

1. x is in the domain of g and
2. $g(x)$ is in the domain of f .

Forming Composite Functions

- 1: Given $f(x) = 5x + 2$ and $g(x) = 3x - 4$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

Inverse Functions

Let f and g be two functions such that

$$f(g(x)) = x \text{ for every } x \text{ in the domain of } g, \text{ and}$$

$$g(f(x)) = x \text{ for every } x \text{ in the domain of } f.$$

The function g is the inverse of the function f , and is denoted by f^{-1} (read "f-inverse"). Thus $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. The domain of f is equal to the range of f^{-1} and vice versa.

Verifying Inverse Functions

- 2: Verify that each function is the inverse of the other:

$$f(x) = 6x \text{ and } g(x) = \frac{x}{6}$$

3: Verify that each function is the inverse of the other.

$$f(x) = 4x + 9 \text{ and } g(x) = \frac{x - 9}{4}$$

Finding the Inverse of a Function

The equation for the inverse of a function can be found as follows:

1. Replace $f(x)$ with y in the equation for $f(x)$.
2. Interchange x and y .
3. Solve for y . If this equation does not define y as a function of x , the function f does not have an inverse function and this procedure ends. If this equation does define y as a function of x , the function f has an inverse function.
4. If f has an inverse function, replace y in step 3 with $f^{-1}(x)$. We can verify our result by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

4: Find the inverse of $f(x) = 6x + 3$

5: Find the inverse of $f(x) = (x+1)^3$

6: Find the inverse of $f(x) = x^3 - 4$.

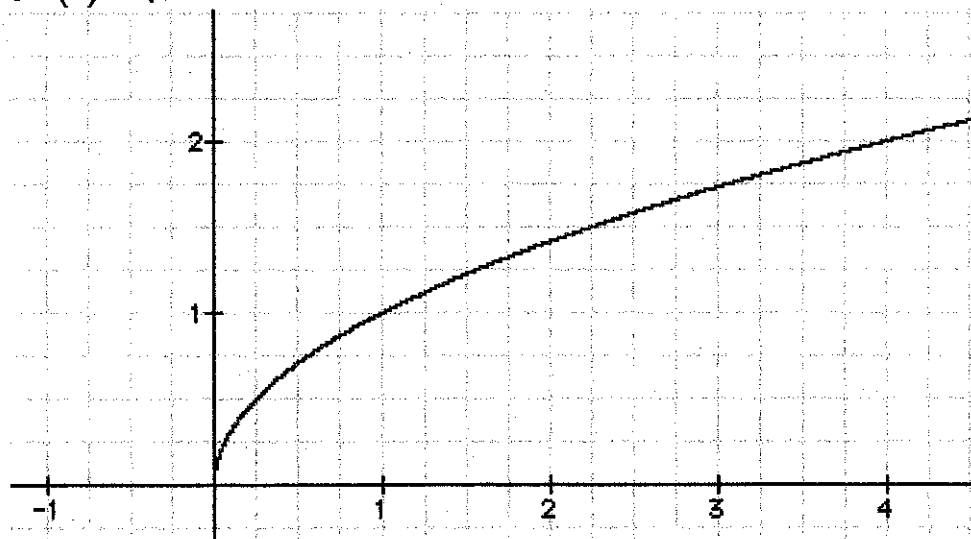
The Horizontal Line Test and One-to-One Functions

The Horizontal Line Test for Functions

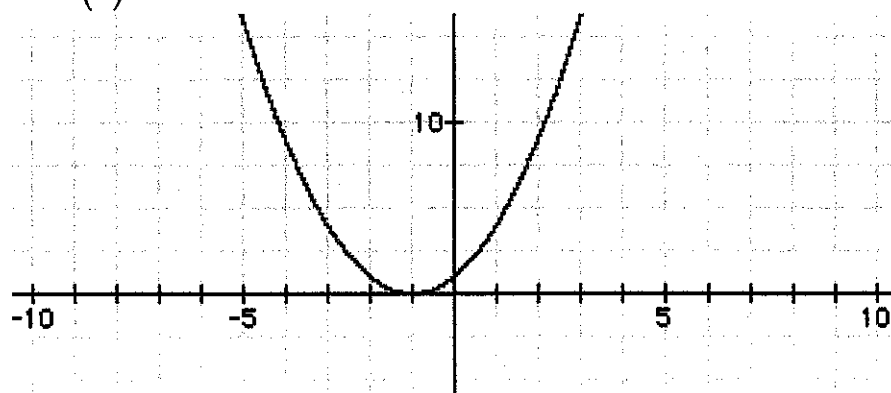
A function f has an inverse that is a function, f^{-1} , if there is no horizontal line that intersects the graph of the function f at more than one point.

7: For each of the following functions, use the given graph of the function and the horizontal line test to determine if the function has an inverse.

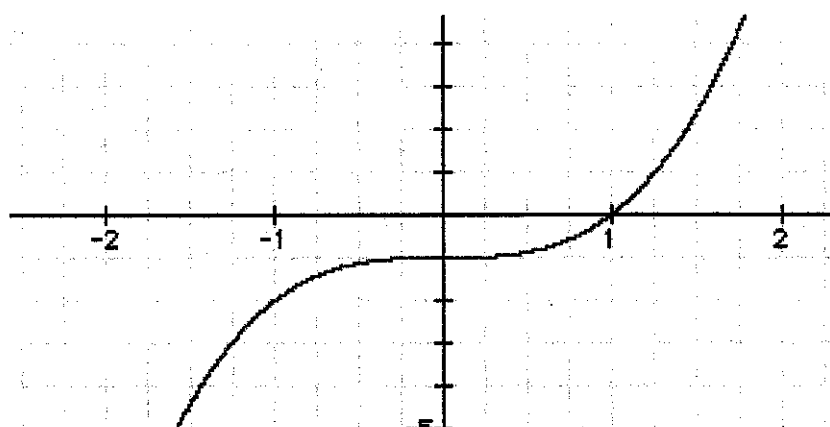
a. $f(x) = \sqrt{x}$



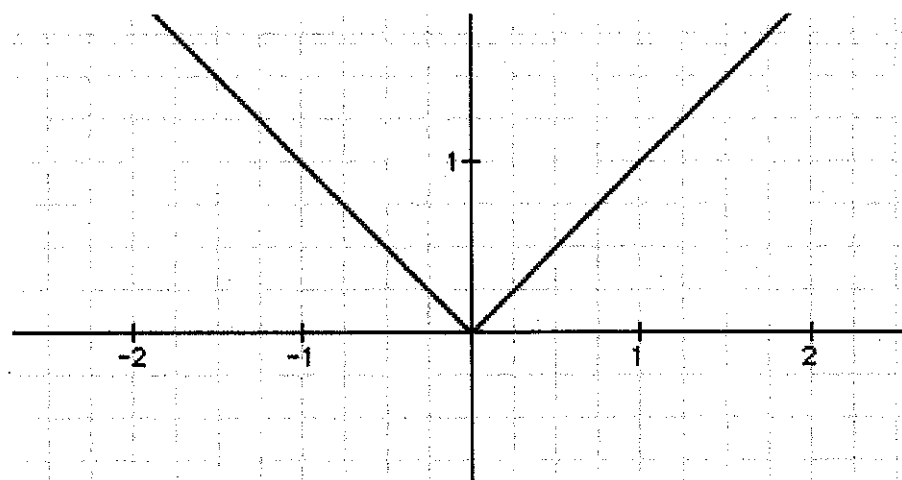
b. $f(x) = x^2 + 2x + 1$



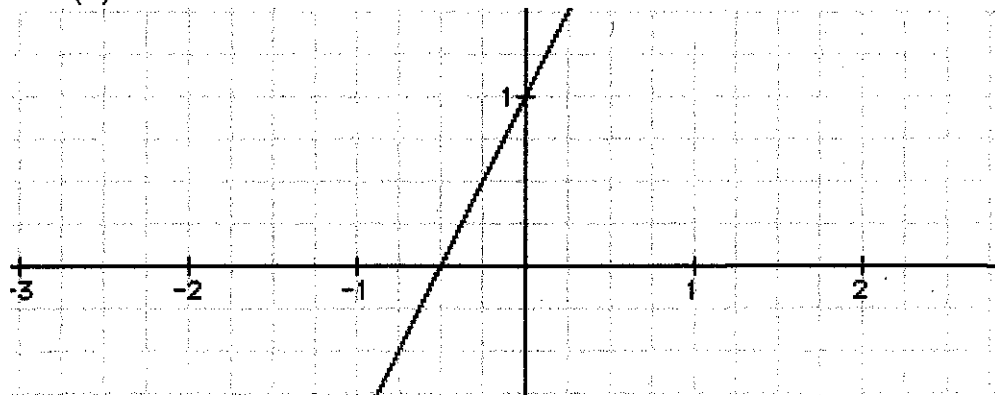
c. $f(x) = x^3 - 1$



d. $f(x) = |x|$



e. $f(x) = 2x + 1$

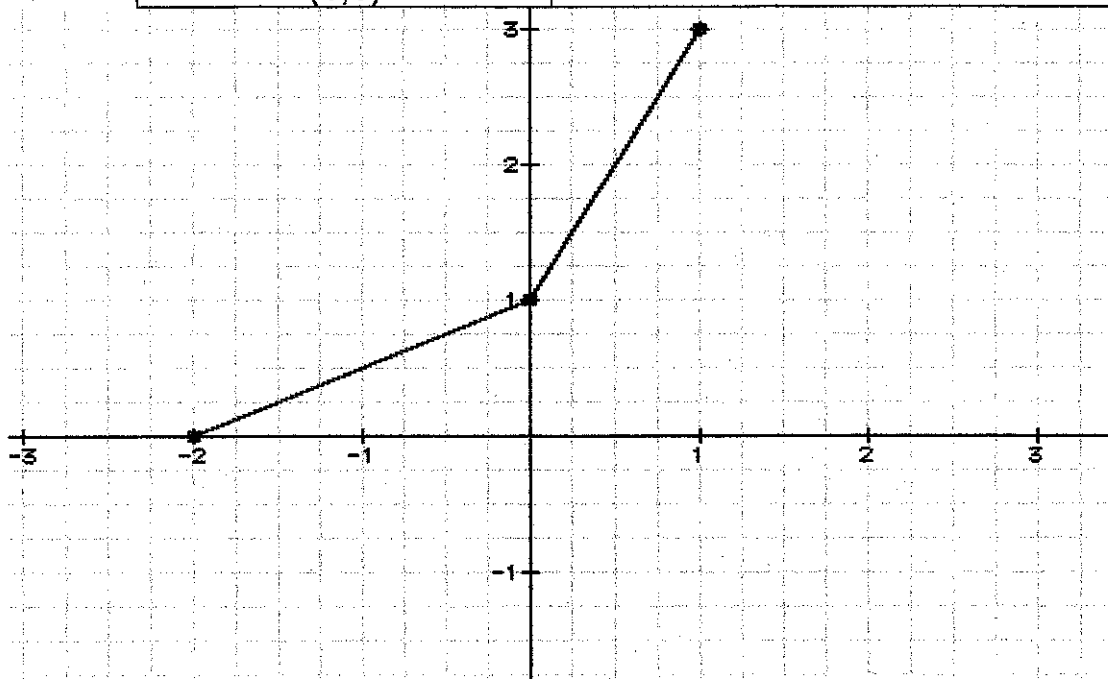


Graphs of f and f^{-1}

The graphs of f and f^{-1} are reflections of one another through the line $y = x$. Points on the graph of f^{-1} can be found by reversing the coordinates of the points on the graph of f .

8: Consider the graph of the function f traced by joining the points given below with straight-line segments. Sketch the graph of f and the graph of f^{-1} .

Points on $y = f(x)$	Points on $y = f^{-1}(x)$
$(-2, 0)$	
$(0, 1)$	
$(1, 3)$	



Parent Function Transformation

1-7 Give the name of the parent function and describe the transformation represented.

1. $g(x) = x^2 - 1$ Name: _____

Transformation: _____

2. $f(x) = 2|x-1|$ Name: _____

Transformation: _____

3. $h(x) = \sqrt{x-2}$ Name: _____

Transformation: _____

4. $g(x) = x^3 + 3$ Name: _____

Transformation: _____

5. $g(x) = \sqrt[3]{x-2}$ Name: _____

Transformation: _____

6. $f(x) = |x+5| - 2$ Name: _____

Transformation: _____

7. $h(x) = \sqrt{x} + 4$ Name: _____

Transformation: _____

#8-12 Identify the domain and range of the function. Describe the transformation from its parent function.

8. $g(x) = 3\sqrt{x}$ Domain : _____ Range : _____

Transformation: _____

9. $h(x) = -x^2 + 1$ Domain : _____ Range : _____

Transformation: _____

10. $h(x) = -|x - 2|$ Domain : _____ Range : _____

Transformation: _____

11. $f(x) = \frac{3}{4}\sqrt{x}$ Domain : _____ Range : _____

Transformation: _____

12. $h(x) = 6(x + 9)^2$ Domain : _____ Range : _____

Transformation: _____

#13 - 17 Given the parent function and a description of the transformation, write the equation of the transformed function, $f(x)$.

13. Absolute value—vertical shift up 5, horizontal shift right 3. _____

14. Square root —vertical compression by $\frac{2}{5}$ _____
 root

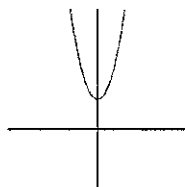
15. Cubic—reflected over the x axis and vertical shift down 2 _____

16. Square root - horizontal shift left 3 _____

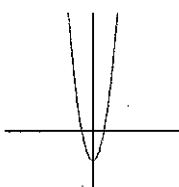
17. Quadratic—vertical compression by .45, horizontal shift left 8. _____

18. Which graph best represents the function $f(x) = 2x^2 - 2$? _____

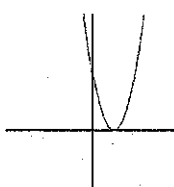
a.



b.



c.



d.

